Confluent Data Reduction for Edge Clique Cover: A Bridge Between Graph Transformation and Kernelization

Hartmut Ehrig Claudia Ermel Falk Hüffner
Rolf Niedermeier Olga Runge

Technische Universität Berlin

2 September 2011

Interaction of data reduction rules

- Kernelizations typically use a set of data reduction rules
- Up to now, little research on interaction of reduction rules

Interaction of data reduction rules

- Kernelizations typically use a set of data reduction rules
- Up to now, little research on interaction of reduction rules

Definition

A set of data reduction rules is called confluent if any order of application yields the same instance.

Why is confluence interesting?

If a kernel is confluent,

- it is “robust”;
- in an implementation, we can optimize for speed of application.

Why is confluence interesting?

If a kernel is confluent,

- it is “robust”;
- in an implementation, we can optimize for speed of application.

If a kernel is not confluent,

- it has “slack”: some orders might lead to worse results;
- investigating this might lead to improved rules.

Why is confluence interesting?

If a kernel is confluent,

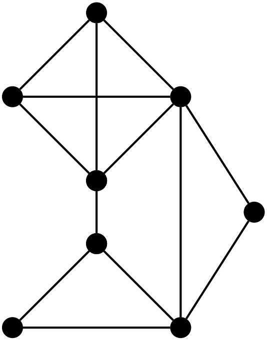
- it is “robust”;
- in an implementation, we can optimize for speed of application.

If a kernel is not confluent,

- it has “slack”: some orders might lead to worse results;
- investigating this might lead to improved rules.

Further, insights on the interaction between rules can lead to faster kernelizations.

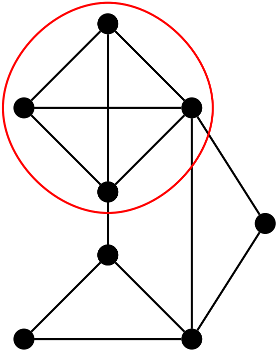
Clique Cover



Clique Cover

Input: An undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.
Question: Is there a set of at most k cliques in G such that each edge in E has both its endpoints in at least one of the selected cliques?

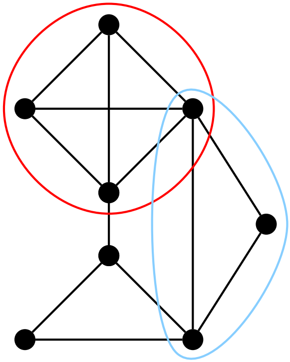
Clique Cover



Clique Cover

Input: An undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.
Question: Is there a set of at most k cliques in G such that each edge in E has both its endpoints in at least one of the selected cliques?

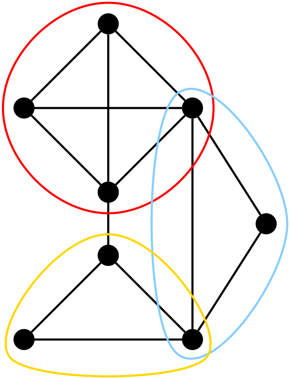
Clique Cover



Clique Cover

Input: An undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.
Question: Is there a set of at most k cliques in G such that each edge in E has both its endpoints in at least one of the selected cliques?

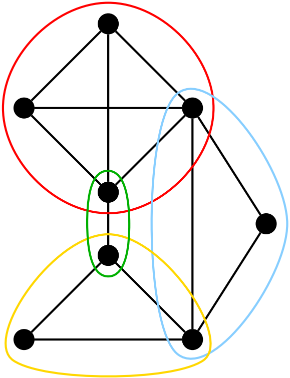
Clique Cover



Clique Cover

Input: An undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.
Question: Is there a set of at most k cliques in G such that each edge in E has both its endpoints in at least one of the selected cliques?

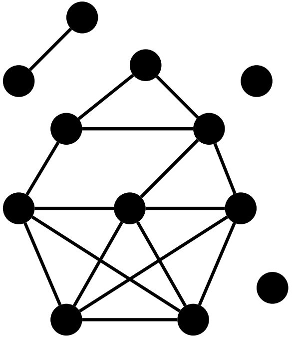
Clique Cover



Clique Cover

Input: An undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.
Question: Is there a set of at most k cliques in G such that each edge in E has both its endpoints in at least one of the selected cliques?

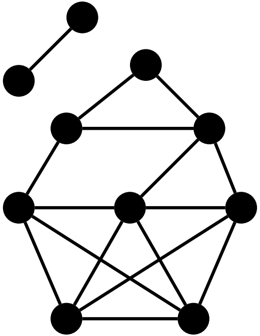
Data reduction for Clique Cover



Rule 1

Delete isolated vertices.

Data reduction for Clique Cover



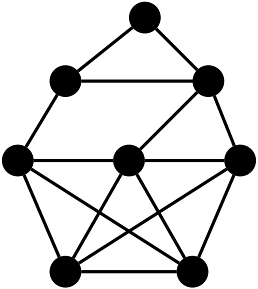
Rule 1

Delete isolated vertices.

Rule 2

Delete isolated edges.

Data reduction for Clique Cover



Rule 1

Delete isolated vertices.

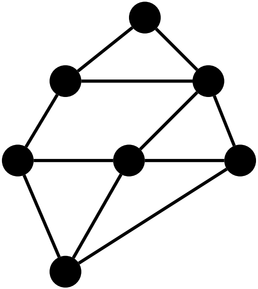
Rule 2

Delete isolated edges.

Rule 3

Delete one of two twins.

Data reduction for Clique Cover



Rule 1

Delete isolated vertices.

Rule 2

Delete isolated edges.

Rule 3

Delete one of two twins.

Kernelization for Clique Cover

Theorem ([Gyárfás 1990, Gramm et al. 2008])

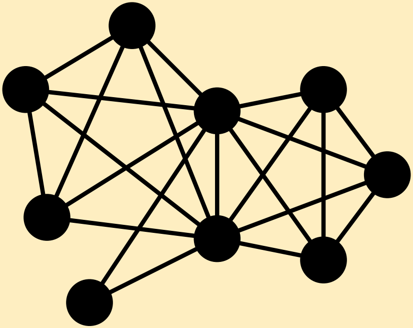
Rules 1 to 3 yield a kernel with at most 2^k *vertices.*

Confluence of Clique Cover kernel

Theorem

Rules 1 to 3 are confluent.

Proof

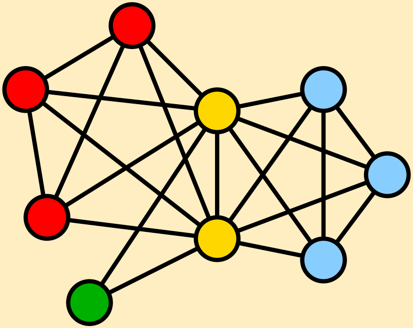


Confluence of Clique Cover kernel

Theorem

Rules 1 to 3 are confluent.

Proof

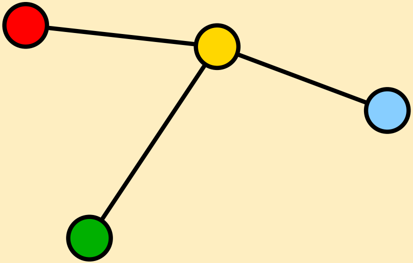


Confluence of Clique Cover kernel

Theorem

Rules 1 to 3 are confluent.

Proof



Confluence of Clique Cover kernel

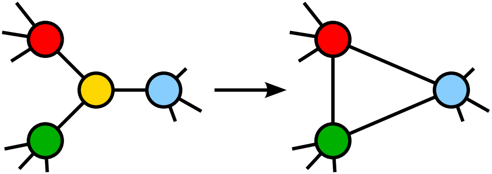
Corollary

A 2^k-vertex kernel for CLIQUE COVER *can be found in linear time.*

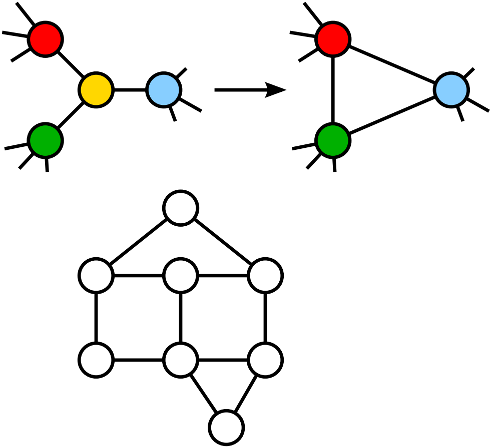
Graph transformation theory

- Started in the early 1970s
- Generalizes Chomsky grammars (on strings) and term rewriting systems (on trees) to graphs
- Used to model operational sematics of changing networks

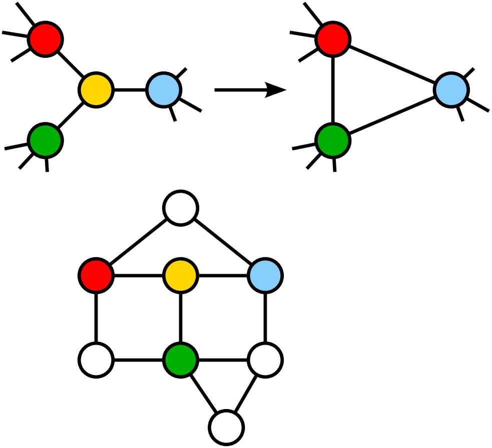
Reduction rules in graph transformation theory



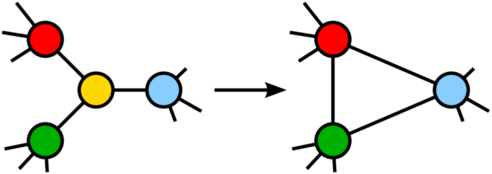
Reduction rules in graph transformation theory

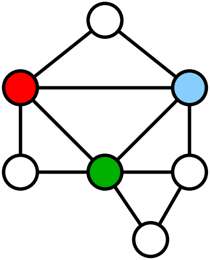


Reduction rules in graph transformation theory

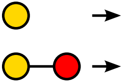


Reduction rules in graph transformation theory

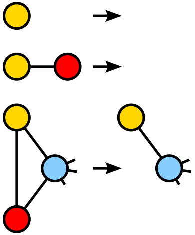




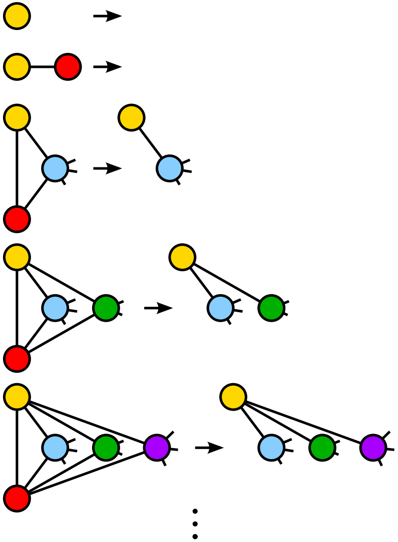
Clique Cover reduction as graph transformation



Clique Cover reduction as graph transformation



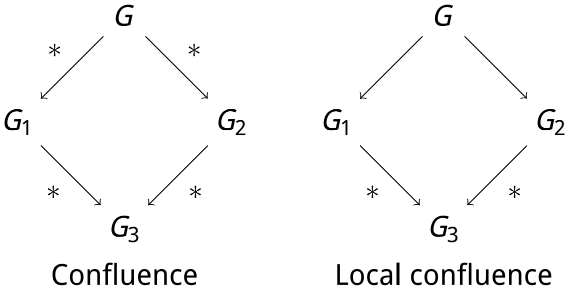
Clique Cover reduction as graph transformation



Local confluence

Newman's lemma [Newman 1942]

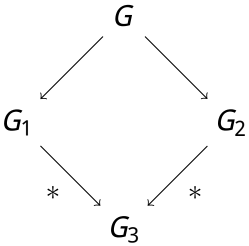
To show confluence of a system of data reduction rules, it is sufficient to show local confluence.



Critical pair analysis

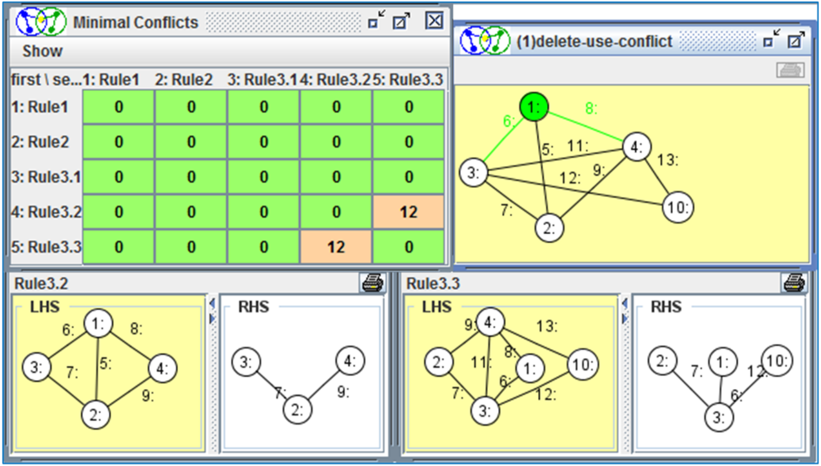
Theorem ([Plump 2005])

To show confluence of a system of data reduction rules on directed graphs, it is sufficient to consider critical pairs, that is, rule applications that conflict and have minimal context.



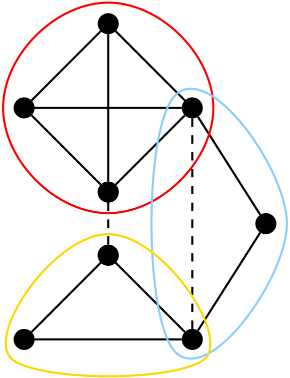
Confluence of critical pair $(G \to G_1, G \to G_2)$

Critical pair analysis with AGG

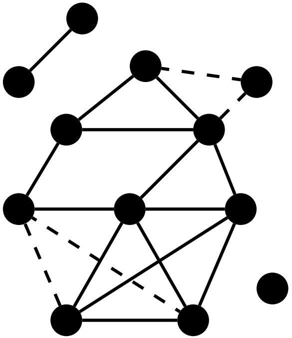


first \ se...	1: Rule1	2: Rule2	3: Rule3.1	4: Rule3.2	5: Rule3.3
1: Rule1	0	0	0	0	0
2: Rule2	0	0	0	0	0
3: Rule3.1	0	0	0	0	0
4: Rule3.2	0	0	0	0	12
5: Rule3.3	0	0	0	12	0

Partial Clique Cover



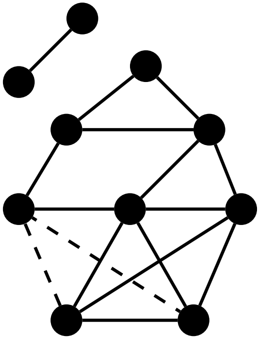
Data reduction for Partial Clique Cover



Rule 4

Delete vertices incident only on covered edges.

Data reduction for Partial Clique Cover



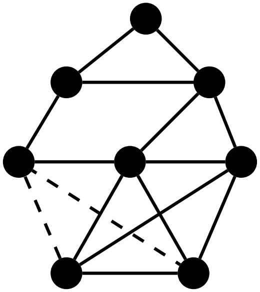
Rule 4

Delete vertices incident only on covered edges.

Rule 5

Delete isolated edges.

Data reduction for Partial Clique Cover



Rule 4

Delete vertices incident only on covered edges.

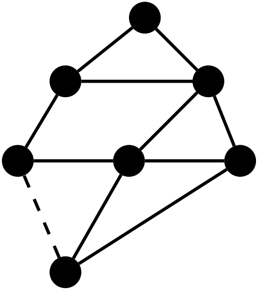
Rule 5

Delete isolated edges.

Rule 6

Delete one of two twins when connections are labelled identically.

Data reduction for Partial Clique Cover



Rule 4

Delete vertices incident only on covered edges.

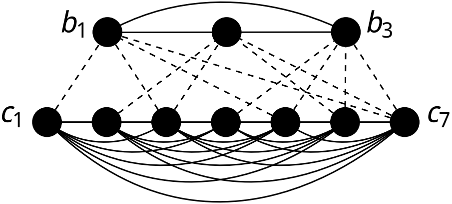
Rule 5

Delete isolated edges.

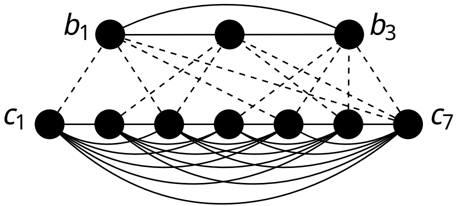
Rule 6

Delete one of two twins when connections are labelled identically.

Kernel for Partial Clique Cover?



Kernel for Partial Clique Cover?



Theorem

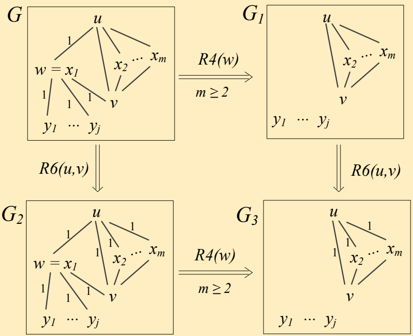
Rules 4 to 6 yield a kernel with at most 2^{k+c} vertices, where c is the number of covered edges.

Confluence of Partial Clique Cover rules

Theorem

Rules 4 to 6 are confluent.

Proof



Future work and open questions

Kernelizations

- Analyze more kernelizations for confluence
- Does it make non-existence proofs easier when only asking for confluent problem kernels?
- Does confluence help subsequent solution strategies that build on top of the kernel?

Future work and open questions

Kernelizations

- Analyze more kernelizations for confluence
- Does it make non-existence proofs easier when only asking for confluent problem kernels?
- Does confluence help subsequent solution strategies that build on top of the kernel?

Graph transformation theory

- Extend critical pair theory to undirected graphs
- Extend critical pair theory to rule schemes
- Extend software tools with this

Future work and open questions

Clique Cover

- Is PARTIAL CLIQUE COVER in FPT wrt. k?
- If so, does it have a singly-exponential kernel wrt. k?