Stable Roommate Problem with Diversity Preferences

Extended Abstract

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ABSTRACT
In the multidimensional stable roommate problem, agents have to be allocated to rooms and have preferences over sets of potential roommates. We study the complexity of finding good allocations of agents to rooms under the assumption that agents have diversity preferences [13]: each agent belongs to one of the two types (e.g., juniors and seniors, artists and engineers), and agents’ preferences over rooms depend solely on the fraction of agents of their own type among their potential roommates. We consider various solution concepts for this setting, such as core and exchange stability, Pareto optimality and envy-freeness. On the negative side, we prove that envy-free, core stable or (strongly) exchange stable outcomes may fail to exist and that the associated decision problems are NP-complete. On the positive side, we show that these problems are in FPT with respect to the room size, which is not the case for the general stable roommate problem.

KEYWORDS
coalition formation; hedonic games; roommate problem

ACM Reference Format:

1 INTRODUCTION
Alice and Bob are planning their wedding. They have agreed on the gift registry and the music to be played, but they still need to decide on the seating plan for the wedding reception. They expect 120 guests, and the reception venue has 20 tables, with each table seating 6 guests. However, this task is far from being easy: e.g., Alice’s great-aunt does not get along with Bob’s family and prefers to share the table with Alice and Bob’s new wedding guests. Moreover, Bob’s friend Charlie wonders if the hapless couple may benefit from consulting the literature on the stable roommate problem. In this problem, the goal is to find a stable mapping of 2n agents into n rooms of size 2, where every agent has a preference relation over her possible roommates [19]. The most popular notion of stability in this context is core stability: no two agents should strictly prefer each other to their current roommate. Another relevant notion is exchange stability: no two agents should want to swap their places. However, for the stable roommate problem, neither core stable nor exchange stable outcomes are guaranteed to exist. Further, while Irving [22] proved that it is possible to decide in quadratic time if an instance of the roommate problem with strict preferences admits a core stable outcome, many other results for core and exchange stability are negative, e.g., it is NP-complete to check whether a roommate problem with ties admits a core stable outcome [26] and NP-complete to check whether a strict roommate problem admits an exchange stable outcome [15]. For the s-dimensional stable roommate problem, where each room has size s, even the core non-emptiness problem for strict preferences is NP-complete for s ³ 3 [21, 25]. Other solution concepts considered in the context of the roommate problem are Pareto optimality, where an outcome is called Pareto optimal if there does not exist a different outcome in which all agents are weakly and some strictly better off [3, 24, 27], and envy-freeness, where an outcome is said to be envy-free if no agent wants to take the place of another agent [1, 6, 18].

However, Charlie then notes that Alice and Bob’s problem has additional structure: the invitees can be classified as bride’s family or groom’s family, and it appears that all constraints on seating arrangements can be expressed in terms of this classification: each person only has preferences over the ratio of groom’s relatives and bride’s relatives at her table. Thus, the problem in question is closely related to hedonic diversity games, recently introduced by Bredereck et al. [13]. These are coalition formation games where agents have diversity preferences, i.e., they are partitioned into two groups (say, red and blue), and every agent is indifferent among coalitions with the same ratio of red and blue agents. However, positive results for hedonic diversity games are not directly applicable to the roommate setting: in hedonic games, agents form groups of varying sizes, while the wedding guests have to be split into groups of six. Thereby, the set of feasible allocations and the set of possible deviations change, and the considered solution concepts differ.

In this paper, we investigate the multidimensional stable roommate problem (for arbitrary room size s) with diversity preferences; we refer to the resulting problem as the roommate diversity problem. This model captures important aspects of several real-world group formation scenarios, such as flat-sharing, splitting students into teams for group projects, and seating arrangements at important events. We consider common solution concepts from the literature on the stable roommate problem; for each solution concept, we analyze the complexity of checking if a given outcome is a valid solution, whether the set of solutions is guaranteed to be non-empty, and, if not, how hard it is to check if an instance admits a solution as well as to compute a solution if it exists.

We show that for room size two, every instance of our problem
admits an outcome that is core stable, exchange stable and Pareto
optimal, by presenting a linear-time algorithm that always com-
putes such an outcome. For $s > 2$, we provide counterexamples
showing that core stable, exchange stable or envy-free outcomes
may fail to exist. Moreover, we show that it is computationally
hard to determine whether an instance of the roommate diversity
problem admits a core stable outcome, a strongly core stable out-
come, a strongly exchange stable outcome or an envy-free outcome;
for Pareto optimality, we show that it is not only hard to find a
Pareto optimal outcome, but also to verify whether a given outcome
is Pareto optimal. Many of our hardness proofs exploit the close
relationship between hedonic diversity games and anonymous he-
donic games [7, 11], where agents’ preferences over coalitions are
determined by coalition sizes.

On the positive side, a same-type exchange stable outcome is
guaranteed to exist with the respective property (Gu.), the complexity of deciding if an instance admits such an
outcome (Ex.) and of finding one if it exists (Co.). For all solution concepts, the problem of verifying whether a given outcome
has the desired property is in P except for Pareto optimality, for which this problem is coNP-complete.

2 MODEL

Definition 2.1. A roommate diversity problem with room size $s$ is
a quadruple $G = (R, B, s, (\succeq_i)_{i \in R \cup B})$ with $N = R \cup B$ and $|N| = k \cdot s$
for some $k \in \mathbb{N}$. The preference relation $\succeq_i$ of each agent $i \in N$ is a
weak order over the set $D = \left\{ \frac{j}{k} : j \in [0, s] \right\}$.

We refer to size-$s$ subsets of $N$ as rooms; the quantity $k = \frac{|N|}{s}$
is then the number of rooms. An outcome of $G$ is a partition of all
agents into $k$ rooms $\pi = \{C_1, \ldots, C_k\}$ such that $|C_i| = s$ for all
$i \in [k]$. Given a room $C \subseteq N$, let $\theta(C)$ denote the fraction of red
agents in $C$, i.e., $\theta(C) = \frac{|C \cap R|}{|C|}$; we say that $C$ is of fraction $\theta(C)$.

For each agent $i \in N$, we interpret her preference relation $\succeq_i$
over $D$ as her preferences over the fraction of red agents in her
room; for instance, $\frac{2}{5} > \frac{3}{5}$ means that $i$ prefers a room where two
out of five agents are red to a room where three out of five agents
are red. Thereby, $\succeq_i$ induces agent $i$’s preferences over all possible
rooms she can be part of. The preference relation of $i \in N$ is said
to be dichotomous if there exists a partition of $D$ into $D^+$ and $D^-$ so
that $i$ is indifferent between elements from the same set but strictly
prefers all elements from $D^+$ to all elements from $D^-$.

In this work, we consider a number of well-known solution
concepts, such as core and exchange stability, Pareto optimality,
and envy-freeness (see Sec. 1), as well as strong core stability (an
outcome is in the strong core if there is no group of $k$ agents such
that each member of the group weakly prefers it to their current
room, and for some agents this preference is strict) and strong
exchange stability (an outcome is strongly exchange stable if there
is no pair of agents that weakly prefer to switch and for at least one
of them this preference is strict). Yet another interesting variant is
same-type exchange stability, where we only allow swaps between
agents of the same type.

3 CONTRIBUTION

We show that for room size two, every instance of our problem
admits an outcome that is core stable, exchange stable and Pareto
optimal, by presenting a linear-time algorithm that always com-
putes such an outcome. For $s > 2$, we provide counterexamples
showing that core stable, exchange stable or envy-free outcomes
may fail to exist. Moreover, we show that it is computationally

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Table 1: For each solution concept and preference restriction, we indicate whether every instance satisfying this restriction is
guaranteed to admit an outcome with the respective property (Gu.), the complexity of deciding if an instance admits such an
outcome (Ex.) and of finding one if it exists (Co.). For all solution concepts, the problem of verifying whether a given outcome
has the desired property is in P except for Pareto optimality, for which this problem is coNP-complete.

4 RELATED WORK

The stable roommate problem was proposed by Gale and Shapley
[19] and has been studied extensively since then [4, 14, 15, 21–26].
It can be seen as a special case of hedonic coalition formation [11],
where agents have to split into groups (with no prior constraints
on the group sizes) and have preferences over groups that they
can be part of. As there is a number of negative results for the
two-dimensional and multi-dimensional roommate problem, it is
important to identify realistic restrictions on the agents’ prefer-
cences for which the associated computational problems become
tractable. This approach has been successful in the study of the
two-dimensional stable roommate problem [2, 9, 12, 16, 17], as well as
in the context of hedonic games [5, 8, 11]. In particular, we build
on the results of Bredereck et al. [13] and Boehmer and Elkind [10],
who analyze the complexity of finding stable outcomes in hedo-
nic diversity games for several notions of stability, such as Nash
stability, individual stability and core stability.
REFERENCES


