Stable Roommate Problem with Diversity Preferences

Extended Abstract

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ABSTRACT

In the multidimensional stable roommate problem, agents have to be allocated to rooms and have preferences over sets of potential roommates. We study the complexity of finding good allocations of agents to rooms under the assumption that agents have diversity preferences [13]: each agent belongs to one of the two types (e.g., juniors and seniors, artists and engineers), and agents' preferences over rooms depend solely on the fraction of agents of their own type among their potential roommates. We consider various solution concepts for this setting, such as core and exchange stability, Pareto optimality and envy-freeness. On the negative side, we prove that envy-free, core stable or (strongly) exchange stable outcomes may fail to exist and that the associated decision problems are NP-complete. On the positive side, we show that these problems are in FPT with respect to the room size, which is not the case for the general stable roommate problem.

KEYWORDS

coalition formation; hedonic games; roommate problem

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1 INTRODUCTION

Alice and Bob are planning their wedding. They have agreed on the gift registry and the music to be played, but they still need to decide on the seating plan for the wedding reception. They expect 120 guests, and the reception venue has 20 tables, with each table seating 6 guests. However, this task is far from being easy: e.g., Alice's great-aunt does not get along with Bob's family and prefers not to share the table with any of them; on the contrary, Bob's younger brother is keen to meet Alice's family and would be upset if he were stuck with his relatives. After spending an evening trying to find a seating plan that would keep everyone happy, Alice and Bob are on the brink of canceling the wedding altogether.

Bob's friend Charlie wonders if the hapless couple may benefit from consulting the literature on the *stable roommate problem*. In this problem, the goal is to find a stable mapping of $2n$ agents into n rooms of size 2, where every agent has a preference relation over her possible roommates [19]. The most popular notion of stability in this context is *core stability*: no two agents should strictly prefer each other to their current roommate. Another relevant notion is *exchange stability*: no two agents should want to swap their places. However, for the stable roommate problem, neither core stable nor exchange stable outcomes are guaranteed to exist. Further, while Irving [22] proved that it is possible to decide in quadratic time if an instance of the roommate problem with strict preferences admits a core stable outcome, many other results for core and exchange stability are negative, e.g., it is NP-complete to check whether a roommate problem with ties admits a core stable outcome [26] and NP-complete to check whether a strict roommate problem admits an exchange stable outcome [15]. For the *s-dimensional stable roommate problem*, where each room has size s, even the core non-emptiness problem for strict preferences is NP-complete for $s \geq 3$ [21, 25]. Other solution concepts considered in the context of the roommate problem are Pareto optimality, where an outcome is called *Pareto optimal* if there does not exist a different outcome in which all agents are weakly and some strictly better off [3, 24, 27], and envy-freeness, where an outcome is said to be *envy-free* if no agent wants to take the place of another agent [1, 6, 18].

However, Charlie then notes that Alice and Bob's problem has additional structure: the invitees can be classified as bride's family or groom's family, and it appears that all constraints on seating arrangements can be expressed in terms of this classification: each person only has preferences over the ratio of groom's relatives and bride's relatives at her table. Thus, the problem in question is closely related to *hedonic diversity games*, recently introduced by Bredereck et al. [13]. These are coalition formation games where agents have *diversity preferences*, i.e., they are partitioned into two groups (say, *red* and *blue*), and every agent is indifferent among coalitions with the same ratio of red and blue agents. However, positive results for hedonic diversity games are not directly applicable to the roommate setting: in hedonic games, agents form groups of varying sizes, while the wedding guests have to be split into groups of six. Thereby, the set of feasible allocations and the set of possible deviations change, and the considered solution concepts differ.

In this paper, we investigate the multidimensional stable roommate problem (for arbitrary room size s) with diversity preferences; we refer to the resulting problem as the *roommate diversity problem*. This model captures important aspects of several real-world group formation scenarios, such as flat-sharing, splitting students into teams for group projects, and seating arrangements at important events. We consider common solution concepts from the literature on the stable roommate problem; for each solution concept, we analyze the complexity of checking if a given outcome is a valid solution, whether the set of solutions is guaranteed to be non-empty, and, if not, how hard it is to check if an instance admits a solution as well as to compute a solution if it exists.

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	unrestricted			strict			dichotomous		
	Gu.	Ex.	Co.	Gu.	Ex.	Co.	Gu.	Ex.	Co.
Core stability	✗	NPc	NPh	✗	NPc	NPh	✓	-	P
Strong core stability	✗	NPc	NPh	✗	NPc	NPh	✗	NPc	NPh
Same-type exchange stability	✓	-	P	✓	-	P	✓	-	P
Exchange stability	✗	?	?	✗	?	?	?	?	?
Strong exchange stability	✗	NPc	NPh	✗	?	?	✗	NPc	NPh
Pareto optimality	✓	-	NPh	✓	-	P	✓	-	NPh
Envy-freeness	✗	NPc	NPh	✗	NPc	NPh	✗	NPc	NPh

Table 1: For each solution concept and preference restriction, we indicate whether every instance satisfying this restriction is guaranteed to admit an outcome with the respective property (Gu.), the complexity of deciding if an instance admits such an outcome (Ex.) and of finding one if it exists (Co.). For all solution concepts, the problem of verifying whether a given outcome has the desired property is in P except for Pareto optimality, for which this problem is coNP-complete.

2 MODEL

Definition 2.1. A *roommate diversity problem* with room size s is a quadruple $G = (R, B, s, (\succsim_i)_{i \in R \cup B})$ with $N = R \cup B$ and $|N| = k \cdot s$ for some $k \in \mathbb{N}$. The preference relation $\succsim_i$ of each agent $i \in N$ is a weak order over the set $D = \left\{ \frac{j}{s} : j \in [0, s] \right\}$.

We refer to size-s subsets of N as *rooms*; the quantity $k = \frac{|N|}{s}$ is then the number of rooms. An *outcome* of G is a partition of all agents into k rooms $\pi = \{C_1, \dots, C_k\}$ such that $|C_i| = s$ for all $i \in [k]$. Given a room $C \subseteq N$, let $\theta(C)$ denote the fraction of red agents in C, i.e., $\theta(C) = \frac{|C \cap R|}{|C|}$: we say that C *is of fraction* $\theta(C)$.

For each agent $i \in N$, we interpret her preference relation $\succsim_i$ over D as her preferences over the fraction of red agents in her room; for instance, $\frac{2}{5} \succ_i \frac{3}{5}$ means that i prefers a room where two out of five agents are red to a room where three out of five agents are red. Thereby, $\succsim_i$ induces agent i's preferences over all possible rooms she can be part of. The preference relation of $i \in N$ is said to be *dichotomous* if there exists a partition of D into D^+ and D^- so that i is indifferent between elements from the same set but strictly prefers all elements from D^+ to all elements from D^-.

In this work, we consider a number of well-known solution concepts, such as core and exchange stability, Pareto optimality, and envy-freeness (see Sec. 1), as well as strong core stability (an outcome is in the strong core if there is no group of k agents such that each member of the group weakly prefers it to their current room, and for some agents this preference is strict) and strong exchange stability (an outcome is strongly exchange stable if there is no pair of agents that weakly prefer to switch and for at least one of them this preference is strict). Yet another interesting variant is same-type exchange stability, where we only allow swaps between agents of the same type.

3 CONTRIBUTION

We show that for room size two, every instance of our problem admits an outcome that is core stable, exchange stable and Pareto optimal, by presenting a linear-time algorithm that always computes such an outcome. For $s > 2$, we provide counterexamples showing that core stable, exchange stable or envy-free outcomes may fail to exist. Moreover, we show that it is computationally hard to determine whether an instance of the roommate diversity problem admits a core stable outcome, a strongly core stable outcome, a strongly exchange stable outcome or an envy-free outcome; for Pareto optimality, we show that it is not only hard to find a Pareto optimal outcome, but also to verify whether a given outcome is Pareto optimal. Many of our hardness proofs exploit the close relationship between hedonic diversity games and anonymous hedonic games [7, 11], where agents' preferences over coalitions are determined by coalition sizes.

On the positive side, a same-type exchange stable outcome is always guaranteed to exist and can be computed in polynomial time. Also, for some of the solution concepts we consider, we obtain positive results under additional assumptions on agents' preferences: e.g., we can compute a Pareto optimal outcome in polynomial time if agents' preferences are strict. Moreover, all existence questions we consider are in FPT with respect to the room size. We summarize our worst-case complexity results in Table 1.

To the best of our knowledge, our FPT results are among the first positive results for the multidimensional stable roommate problem. Thus, the roommate diversity problem offers an attractive combination of expressive power and computational tractability.

4 RELATED WORK

The stable roommate problem was proposed by Gale and Shapley [19] and has been studied extensively since then [4, 14, 15, 21–26]. It can be seen as a special case of *hedonic coalition formation* [11], where agents have to split into groups (with no prior constraints on the group sizes) and have preferences over groups that they can be part of. As there is a number of negative results for the two-dimensional and multi-dimensional roommate problem, it is important to identify realistic restrictions on the agents' preferences for which the associated computational problems become tractable. This approach has been successful in the study of the two-dimensional stable roommate problem [2, 9, 12, 16, 17], as well as in the context of hedonic games [5, 8, 11]. In particular, we build on the results of Bredereck et al. [13] and Boehmer and Elkind [10], who analyze the complexity of finding stable outcomes in hedonic diversity games for several notions of stability, such as Nash stability, individual stability and core stability.

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