Interval Scheduling and Colorful Independent Sets

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Abstract Numerous applications in scheduling, such as resource allocation or steel manufacturing, reduce to the NP-hard INDEPENDENT SET problem (given an undirected graph and an integer k, find a set of at least k pairwise non-adjacent vertices). Here, one encounters special graph classes like 2-union graphs (edge-wise unions of two interval graphs) and strip graphs (edge-wise unions of an interval graph and a cluster graph), on which INDEPENDENT SET remains NP-hard but admits constant-factor approximations in polynomial time.

We study the parameterized complexity of INDEPENDENT SET on 2-union graphs and on subclasses like strip graphs. Our investigations significantly benefit from a new structural "compactness" parameter of interval graphs and novel problem formulations using vertex-colored interval graphs. Our main contributions are:

1. We show a complexity dichotomy: INDEPENDENT SET is polynomial-time solvable if both input interval graphs are cluster graphs, and NP-hard otherwise.
2. We chart the possibilities and limits of effective polynomial-time preprocessing (also known as kernelization).
3. We extend Halldórsson and Karlsson (2006)'s fixed-parameter algorithm for INDEPENDENT SET on strip graphs parameterized by the structural parameter "maximum number

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of live jobs" to show that the problem (also known as JOB INTERVAL SELECTION) is fixed-parameter tractable with respect to the parameter k and generalize their algorithm from strip graphs to 2-union graphs. Preliminary experiments with random data indicate that JOB INTERVAL SELECTION with up to fifteen jobs and $5 \cdot 10^5$ intervals can be solved optimally in less than five minutes.

Keywords interval graphs · 2-union graphs · strip graphs · job interval selection · parameterized complexity

Bibliography

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