Experiments on Data Reduction for Optimal Domination in Networks

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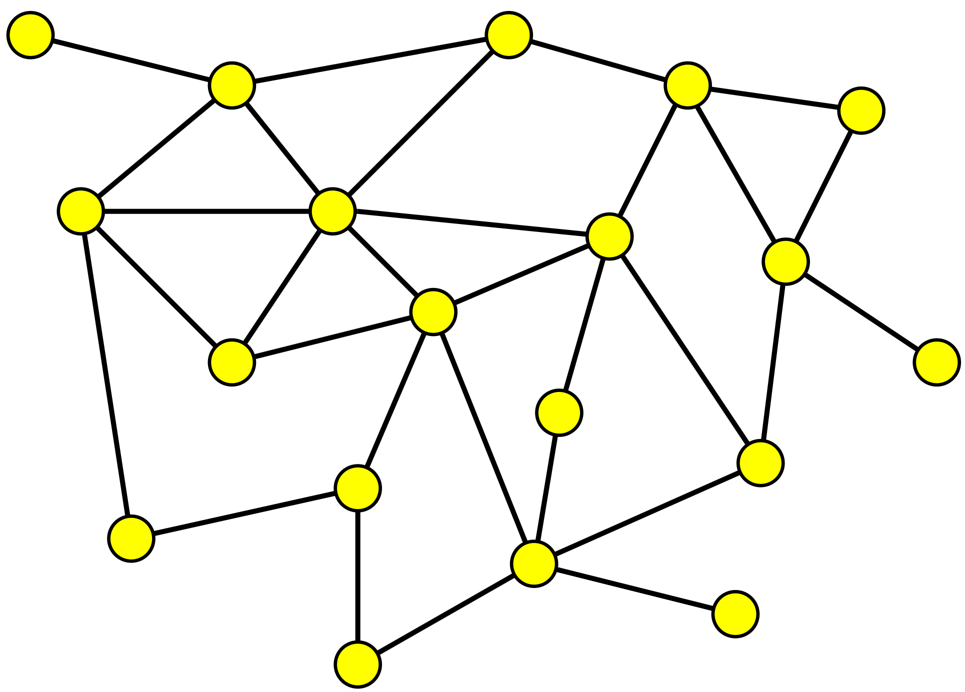
DOMINATING SET

Input: an undirected graph $G = (V, E)$

Question: a subset $V' \subseteq V$ of minimum size such that each vertex in $V - V'$ has a neighbor in V'?



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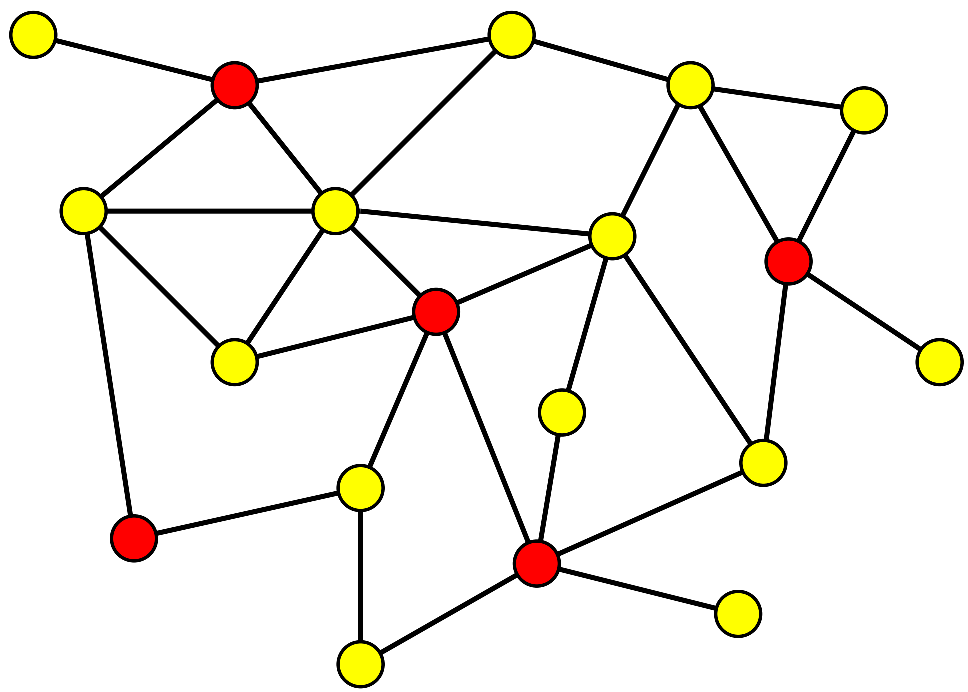

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Question: a subset $V' \subseteq V$ of minimum size such that each vertex in $V - V'$ has a neighbor in V'?

NP-complete!

Applications

Intuition: “Most important vertices in a graph.”

- **facility location**

  e.g., placement of fire-stations or time servers

- **social network analysis**
- **biological network analysis**
- **voting situations**

Topic of this talk

polynomial-time preprocessing

by exploring the local structure

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"simple" reduction rules

to cut away easy parts of the input

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optimal solution

can still be found after data reduction

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experimental results

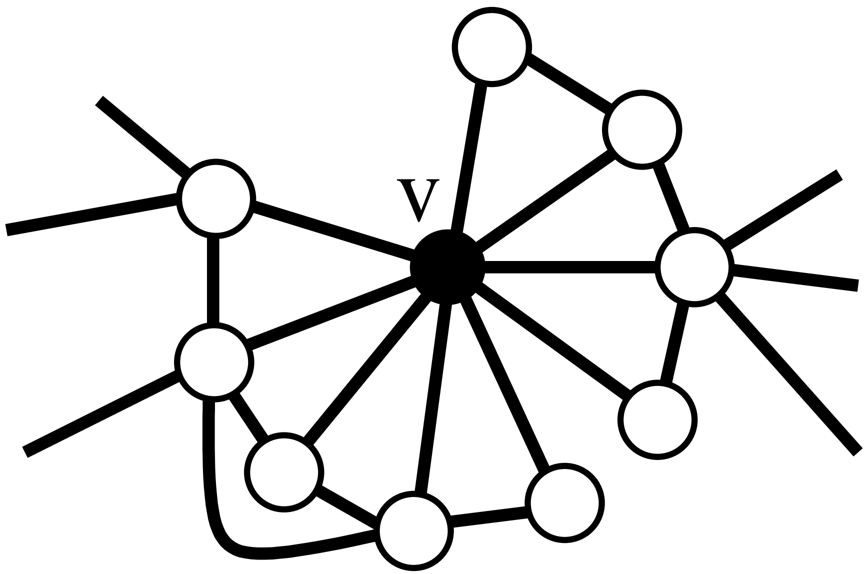
- large internet networks
- directed networks

The Reduction Rules

Reduction Rule 1

Neighborhood $N(v)$ of a single vertex

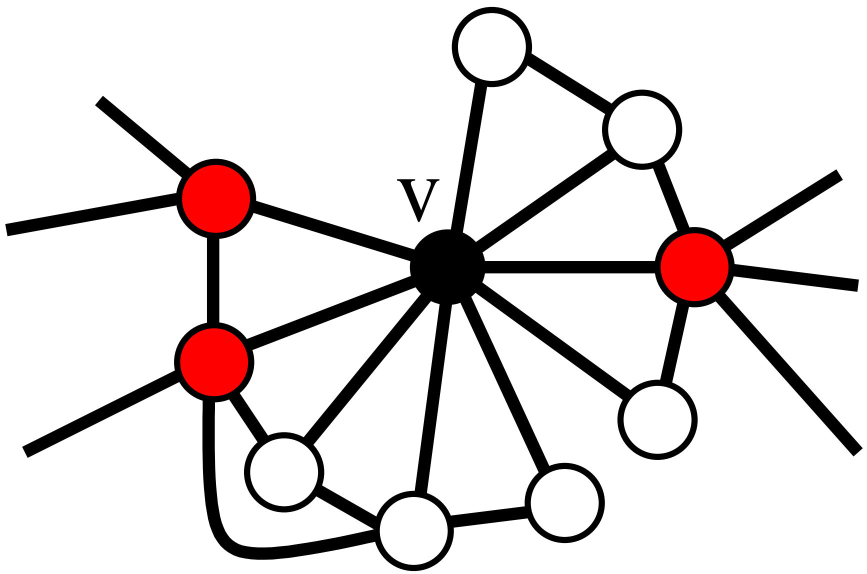
$$N(v) = \ldots$$



Reduction Rule 1

Neighborhood $N(v)$ of a single vertex

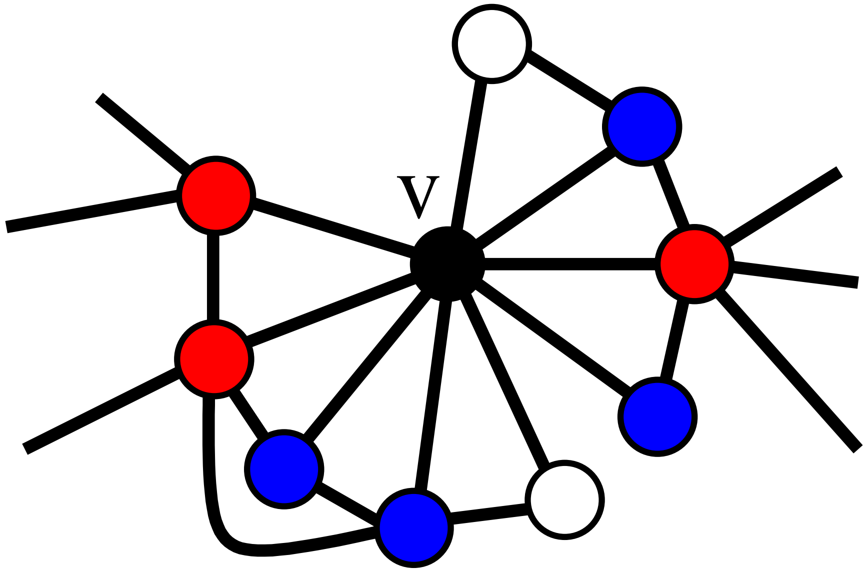
$$N(v) = N_{\text{exit}}(v) \cup \ldots$$



"exit vertices"

Neighborhood $N(v)$ of a single vertex

$$N(v) = N_{\text{exit}}(v) \cup N_{\text{guard}}(v) \cup \ldots$$

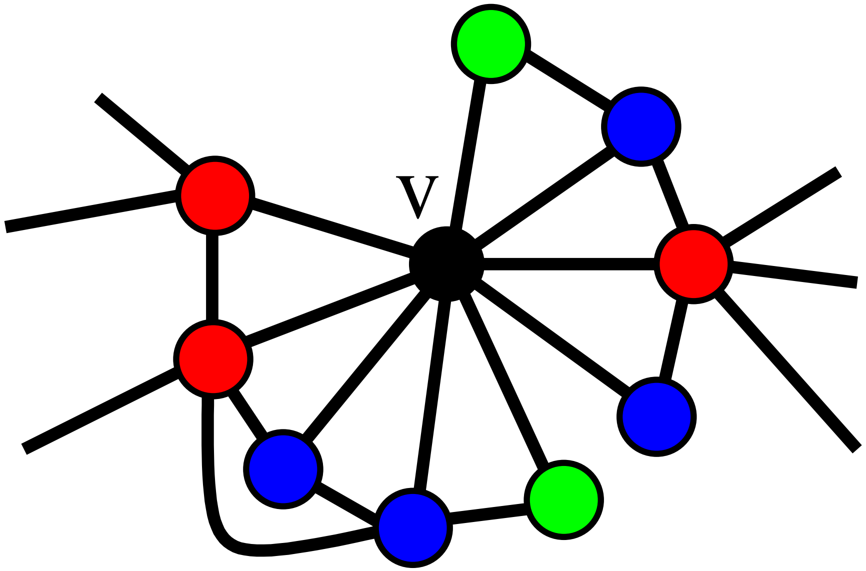


"guard vertices"

Reduction Rule 1

Neighborhood $N(v)$ of a single vertex

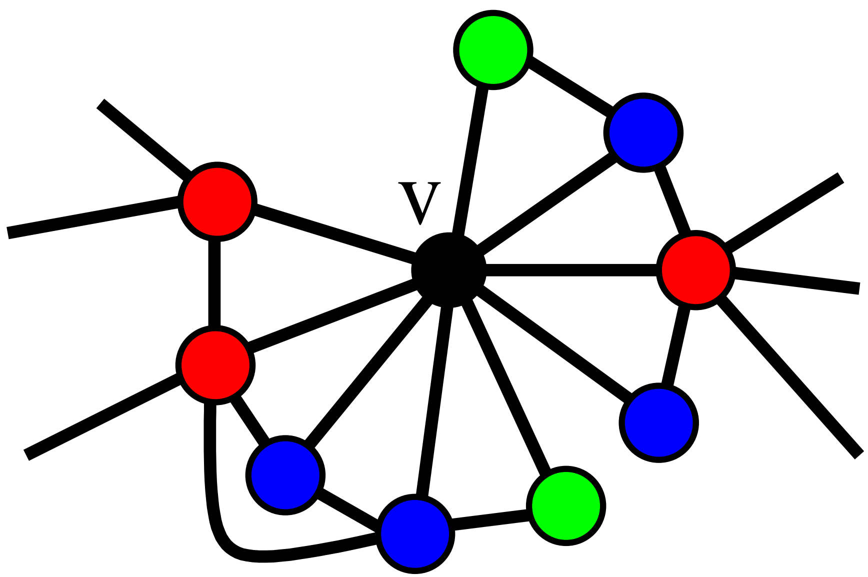
$$N(v) = N_{\text{exit}}(v) \cup N_{\text{guard}}(v) \cup N_{\text{prison}}(v)$$



”prisoner vertices”

Reduction Rule 1

Rule 1: If $N_{\text{prison}}(v) \neq \emptyset$, then choose v.



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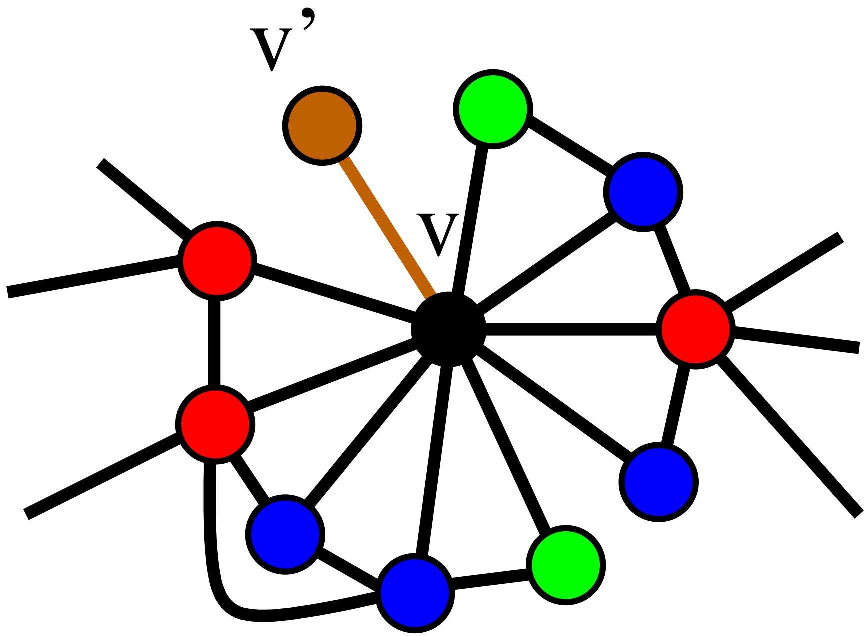

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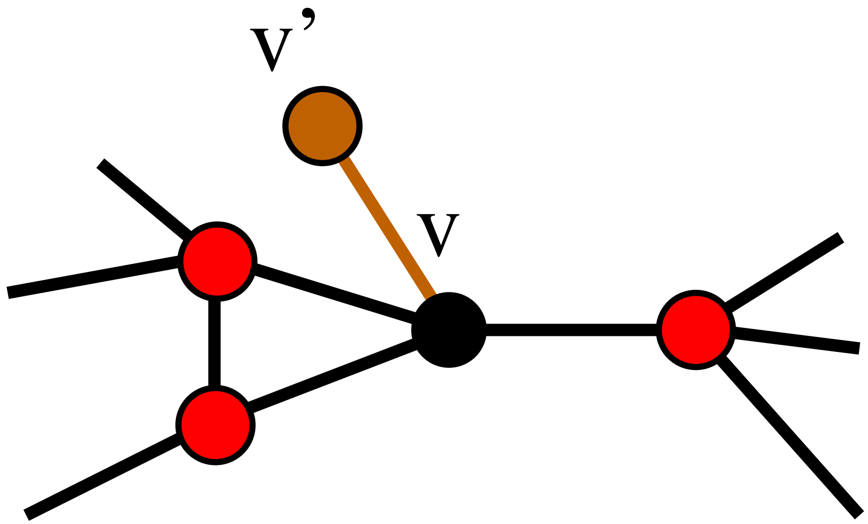
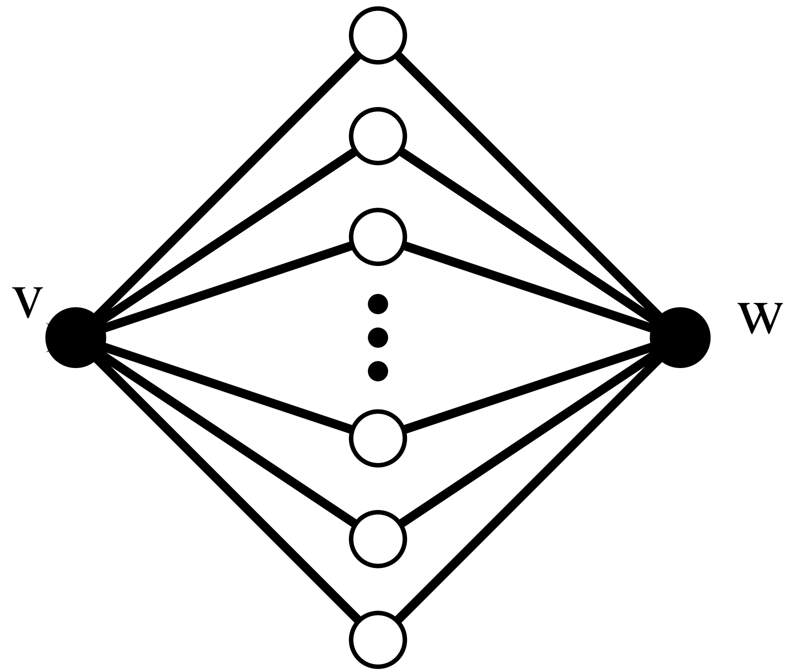


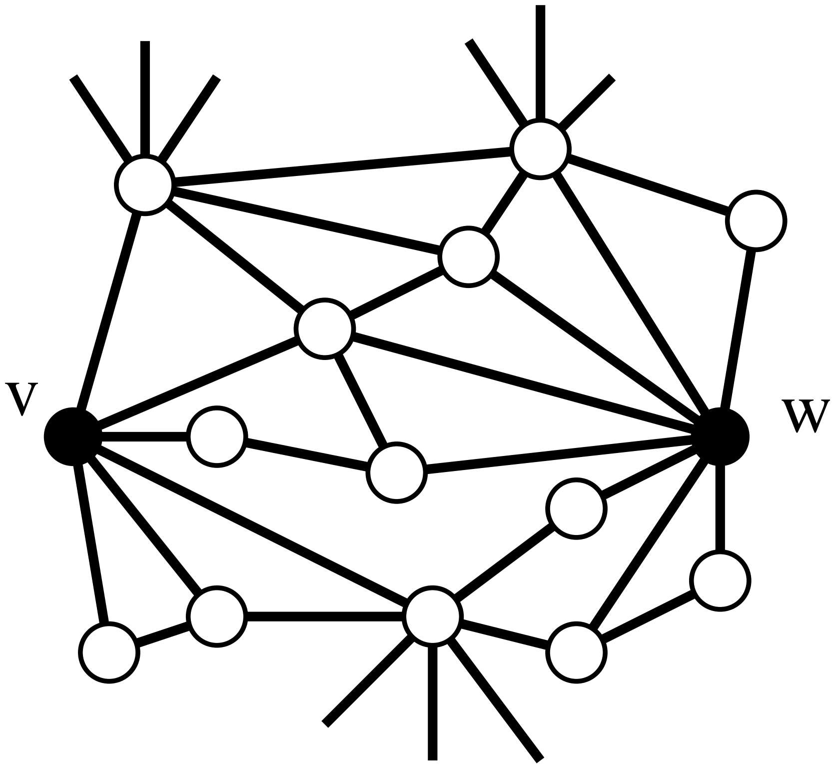
A Hard instance for Rule I



Rule I does not apply

Neighborhood $N(v, w)$ of a pair of vertices

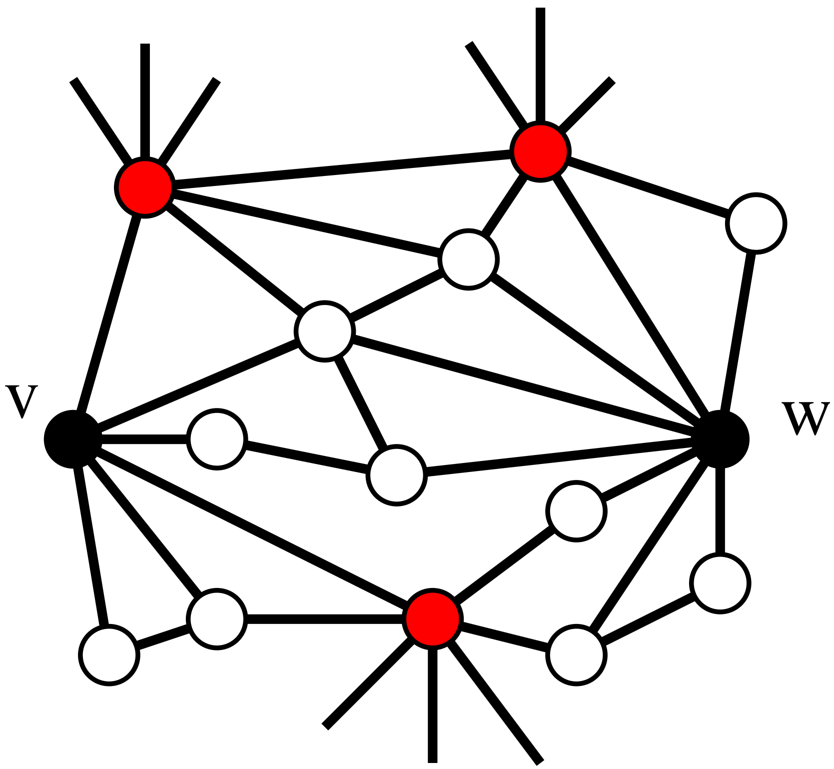
$$N(v, w) = \ldots$$



Reduction Rule 2

Neighborhood $N(v, w)$ of a pair of vertices

$$N(v, w) = N_{\text{exit}}(v, w) \cup \ldots$$



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Reduction Rule 2

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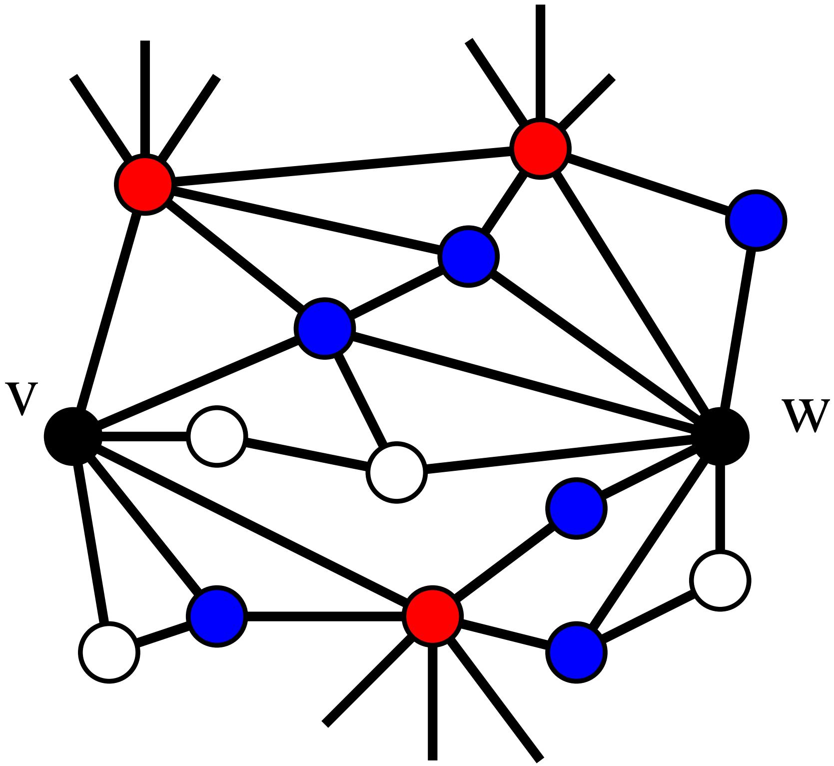
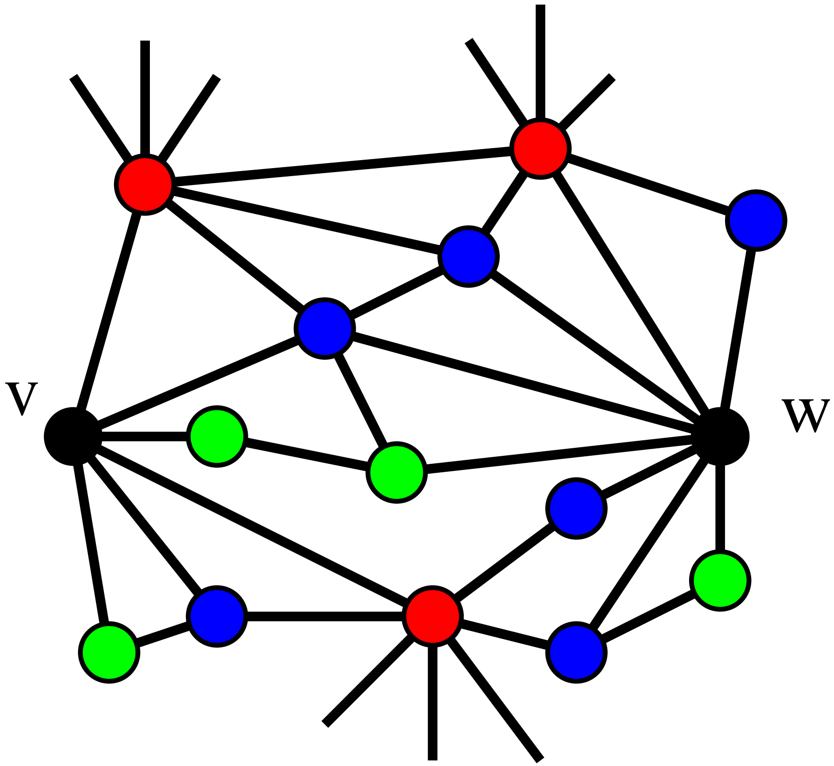


"guard vertices"

Reduction Rule 2

Neighborhood $N(v,w)$ of a pair of vertices

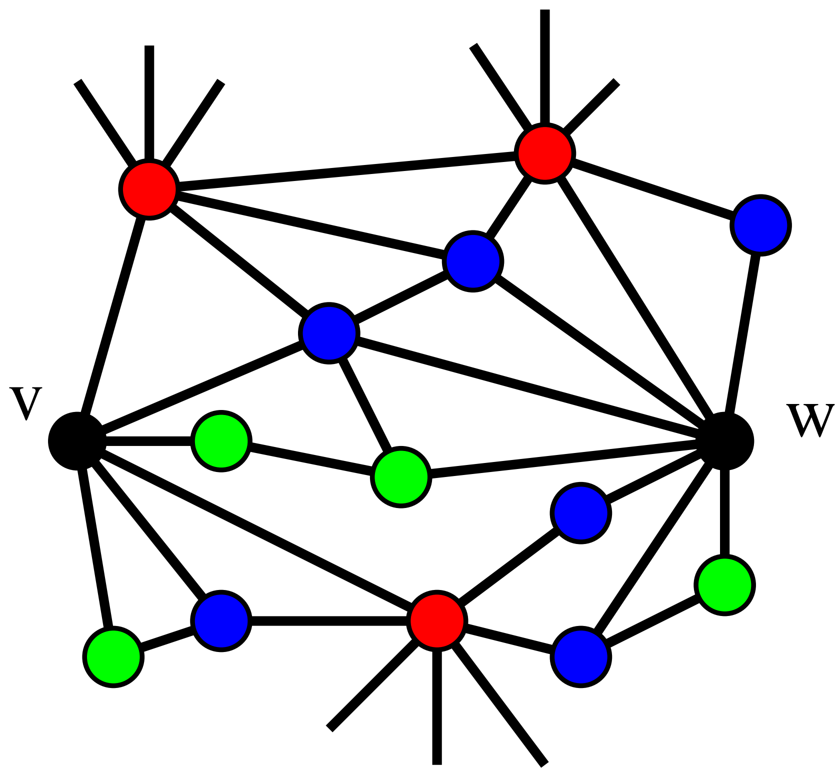
$$N(v,w) = N_{\text{exit}}(v,w) \cup N_{\text{guard}}(v,w) \cup N_{\text{prison}}(v)$$



"prisoner vertices"

Rule 2:

Case 1 If $N_{\text{prison}}(v, w)$ not dominated by a single vertex, then choose v and w.



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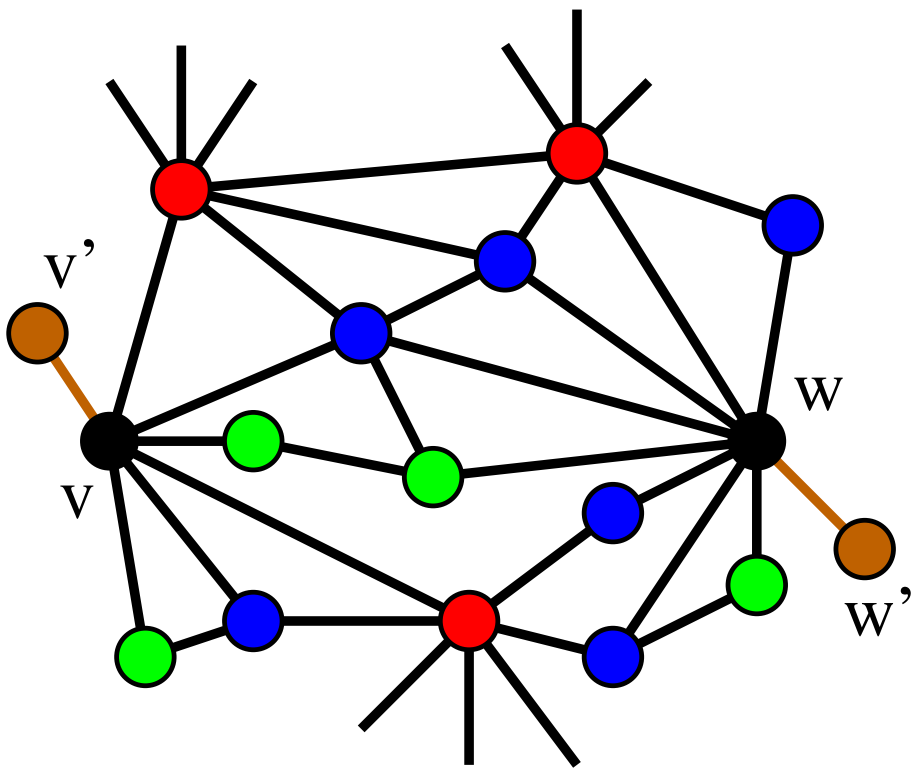

Reduction Rule 2.1

Rule 2:

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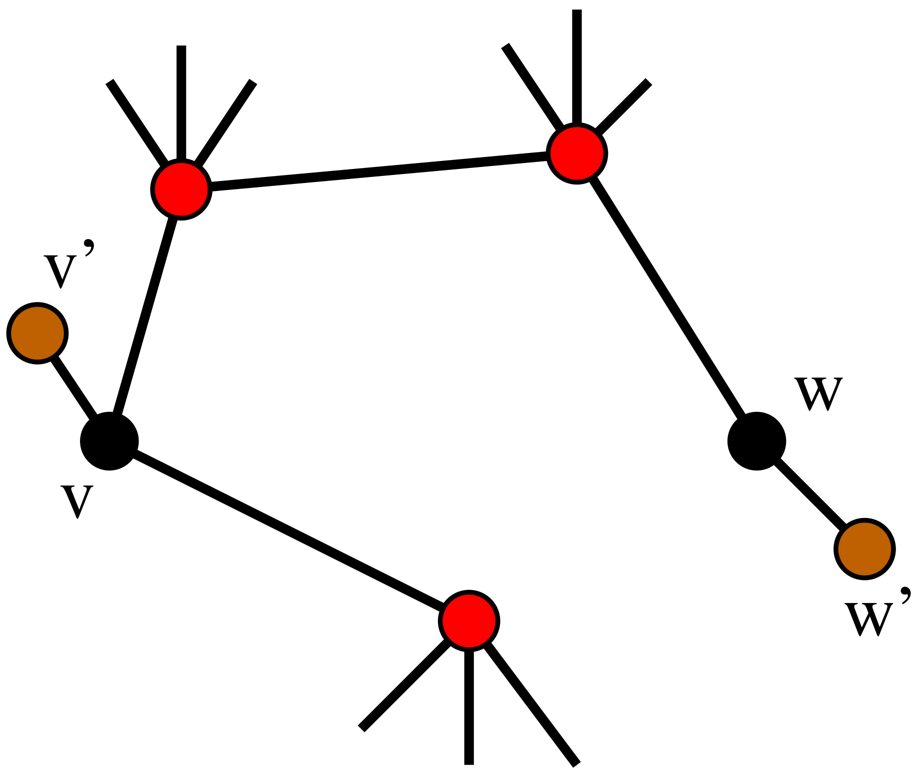
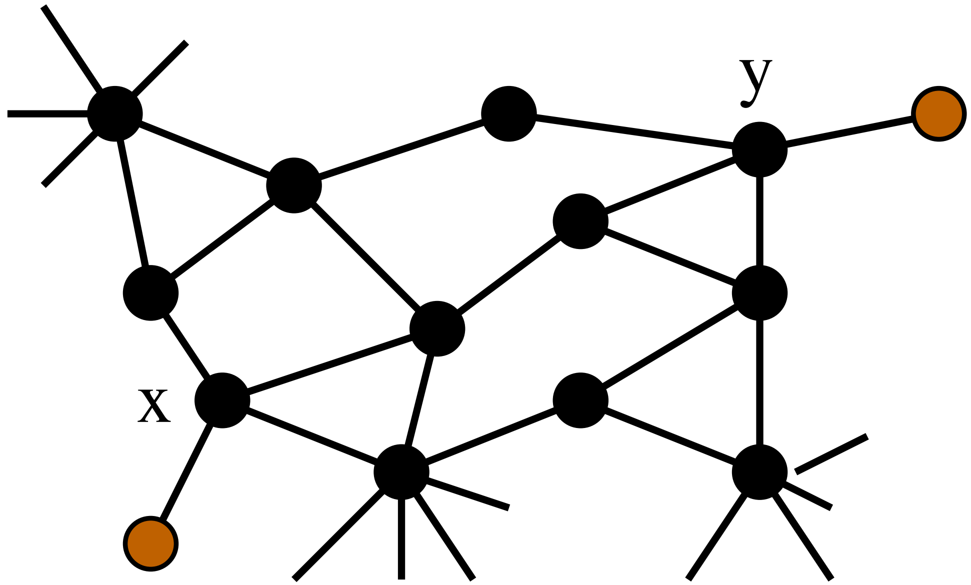


Annotated Dominating Set



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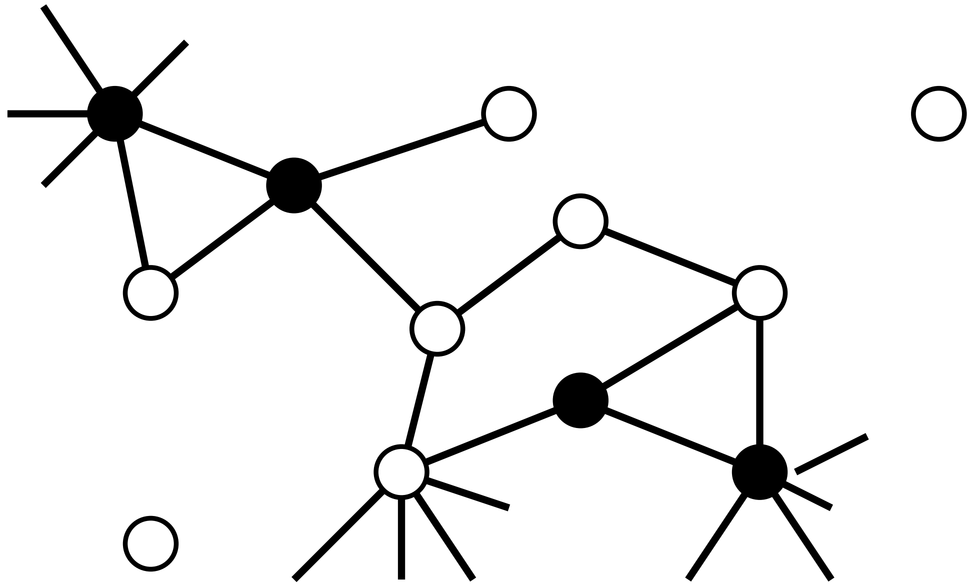
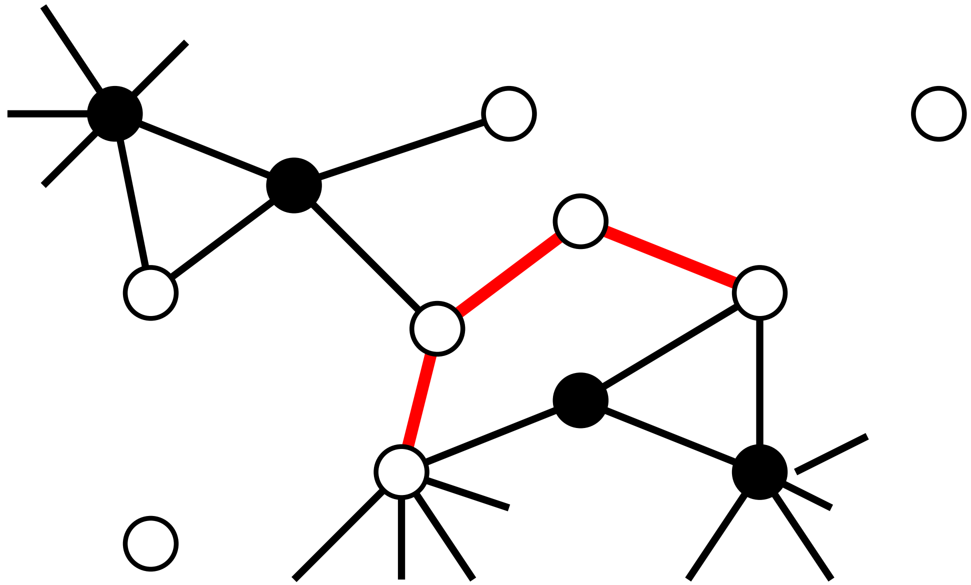


black vertex:....   needs to be dominated

white vertex:....   does **not** need to be dominated

Further Reduction Rules



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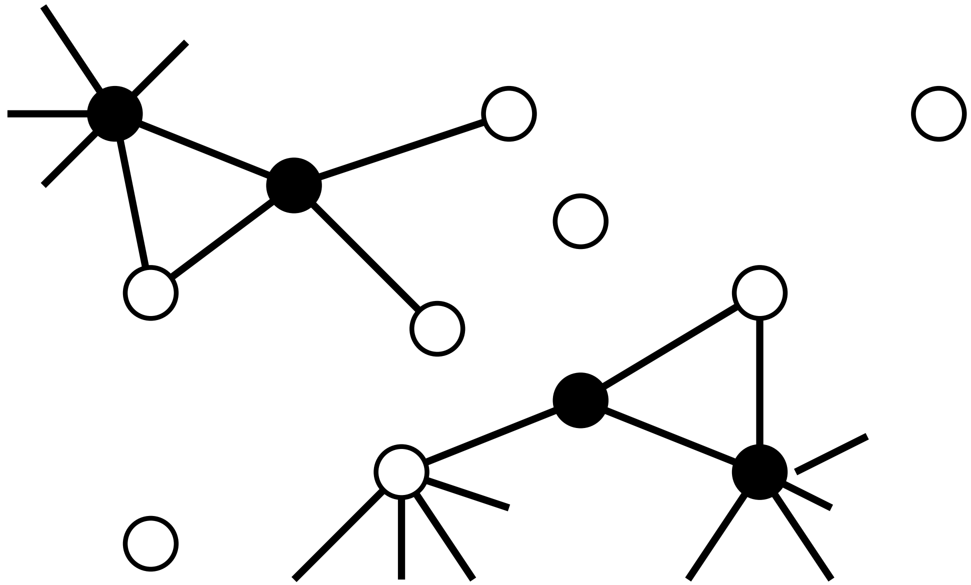

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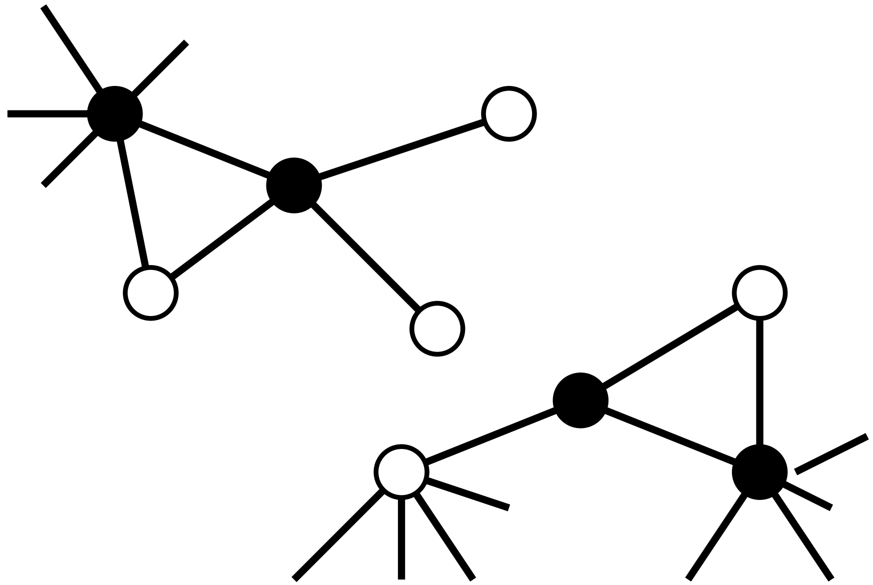

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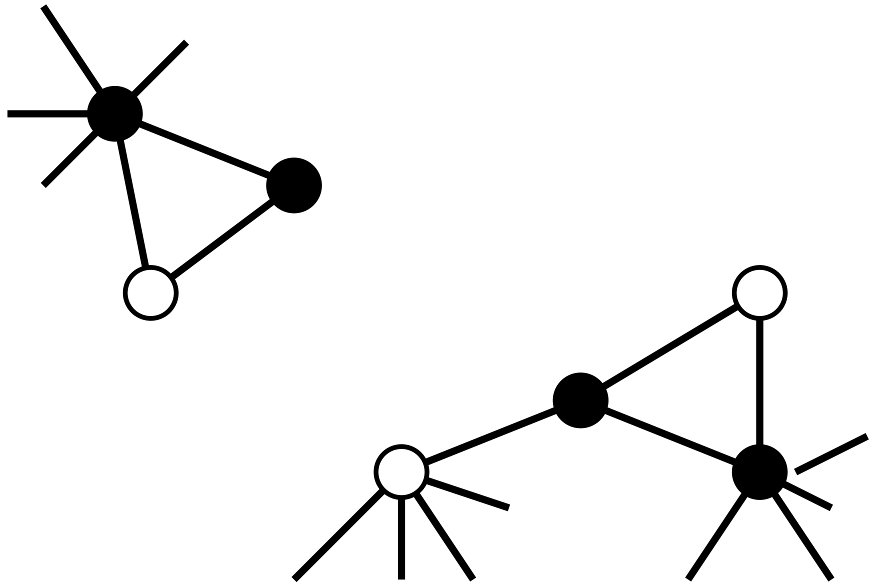

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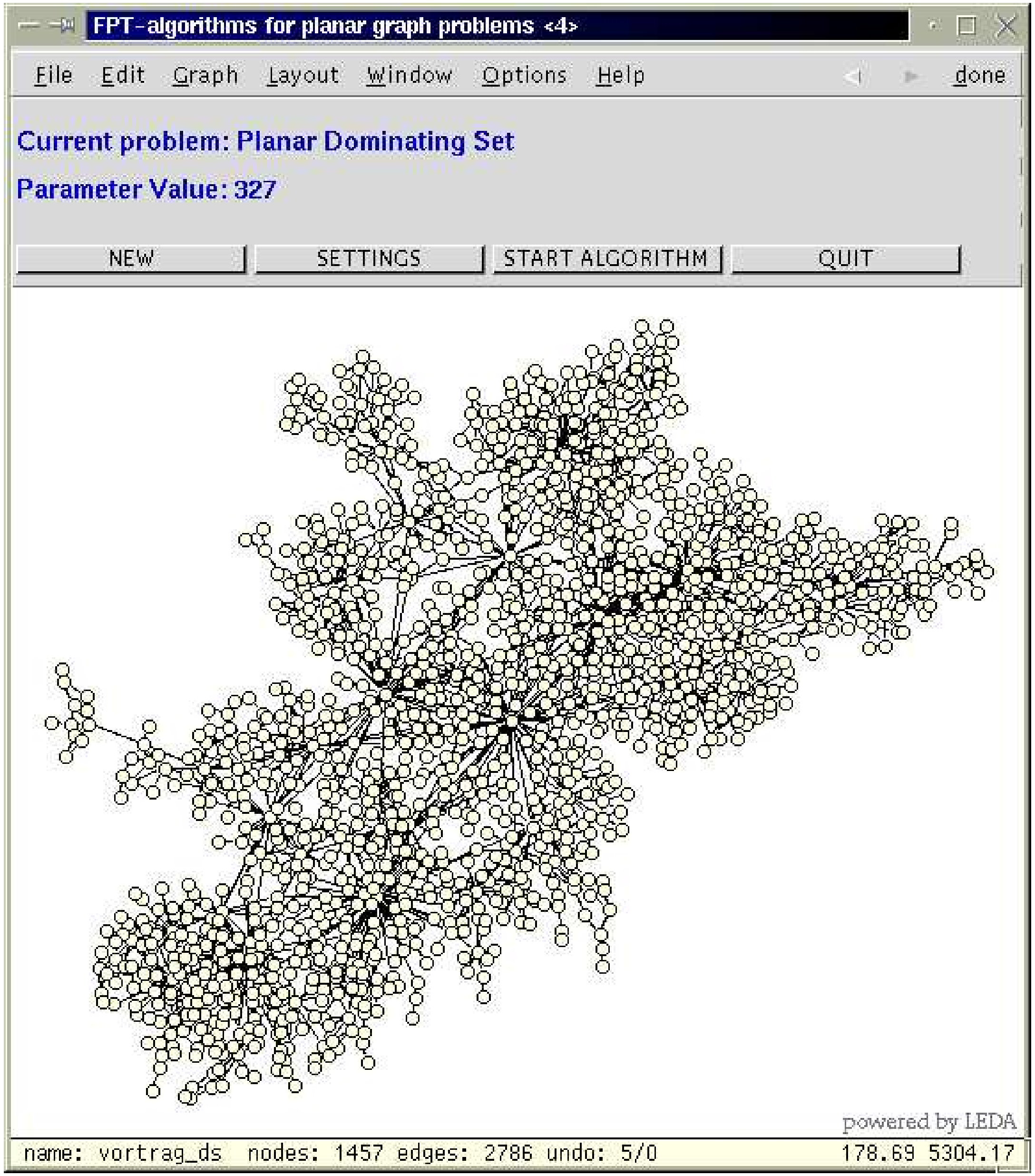
Implementation



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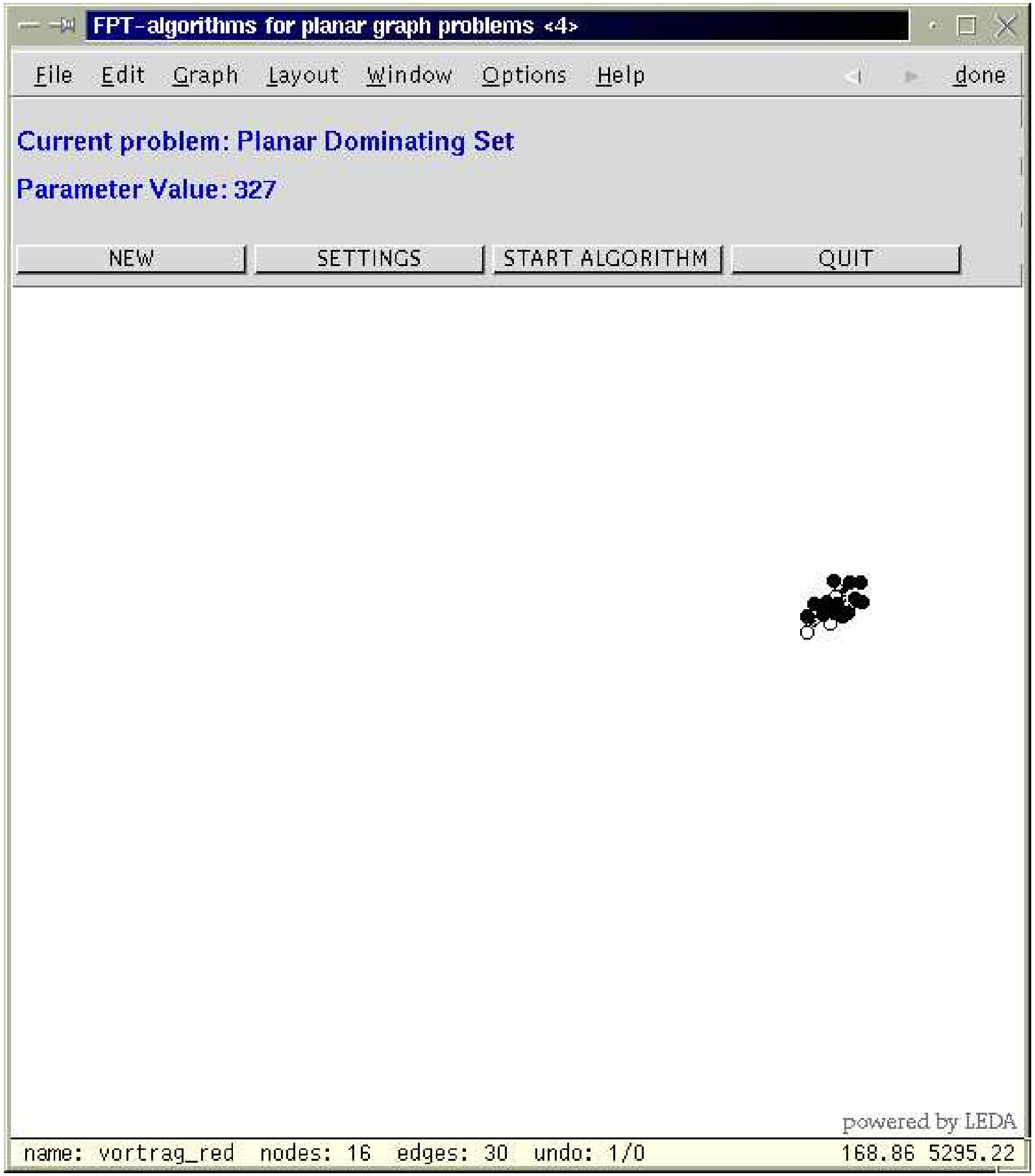

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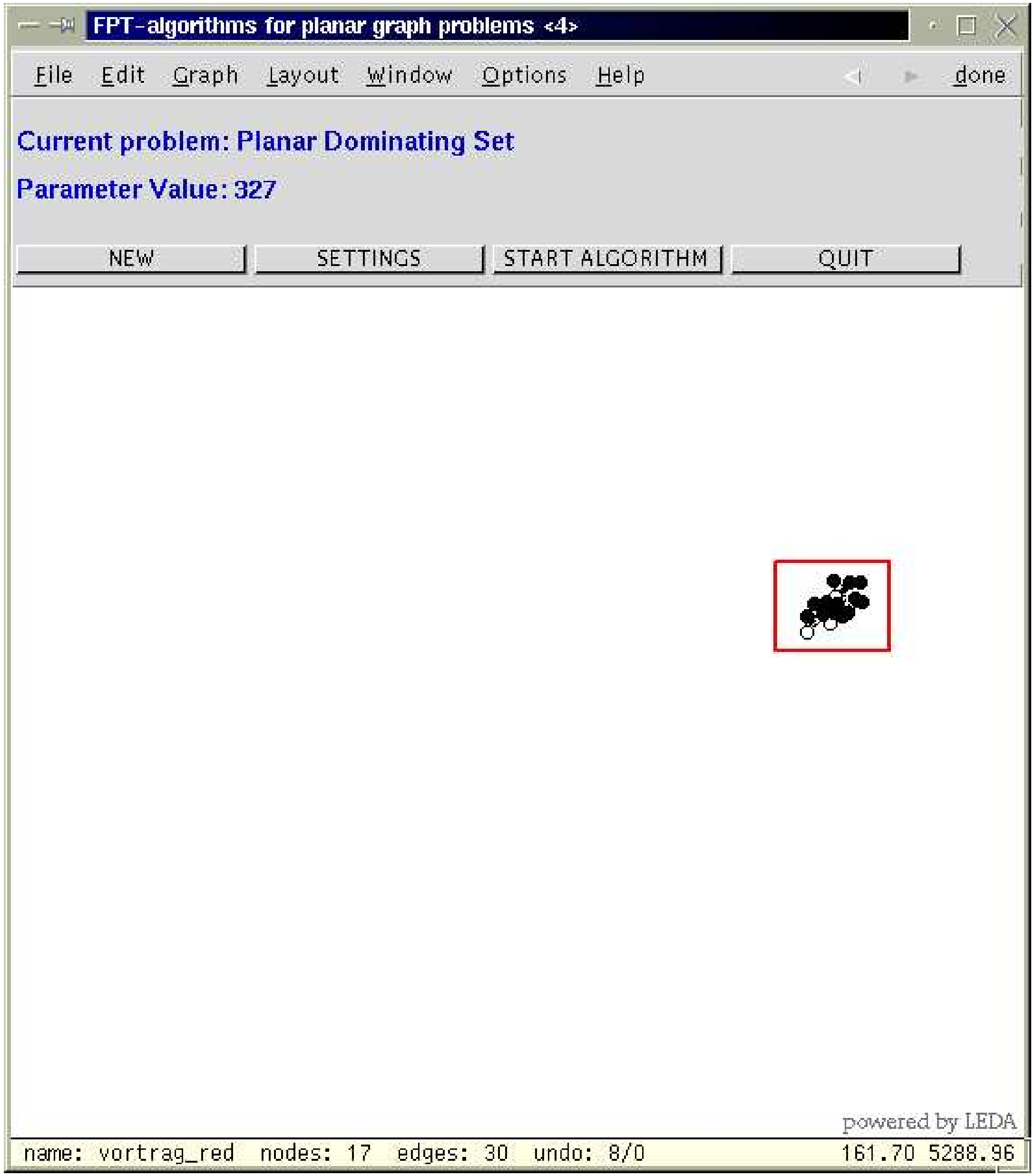

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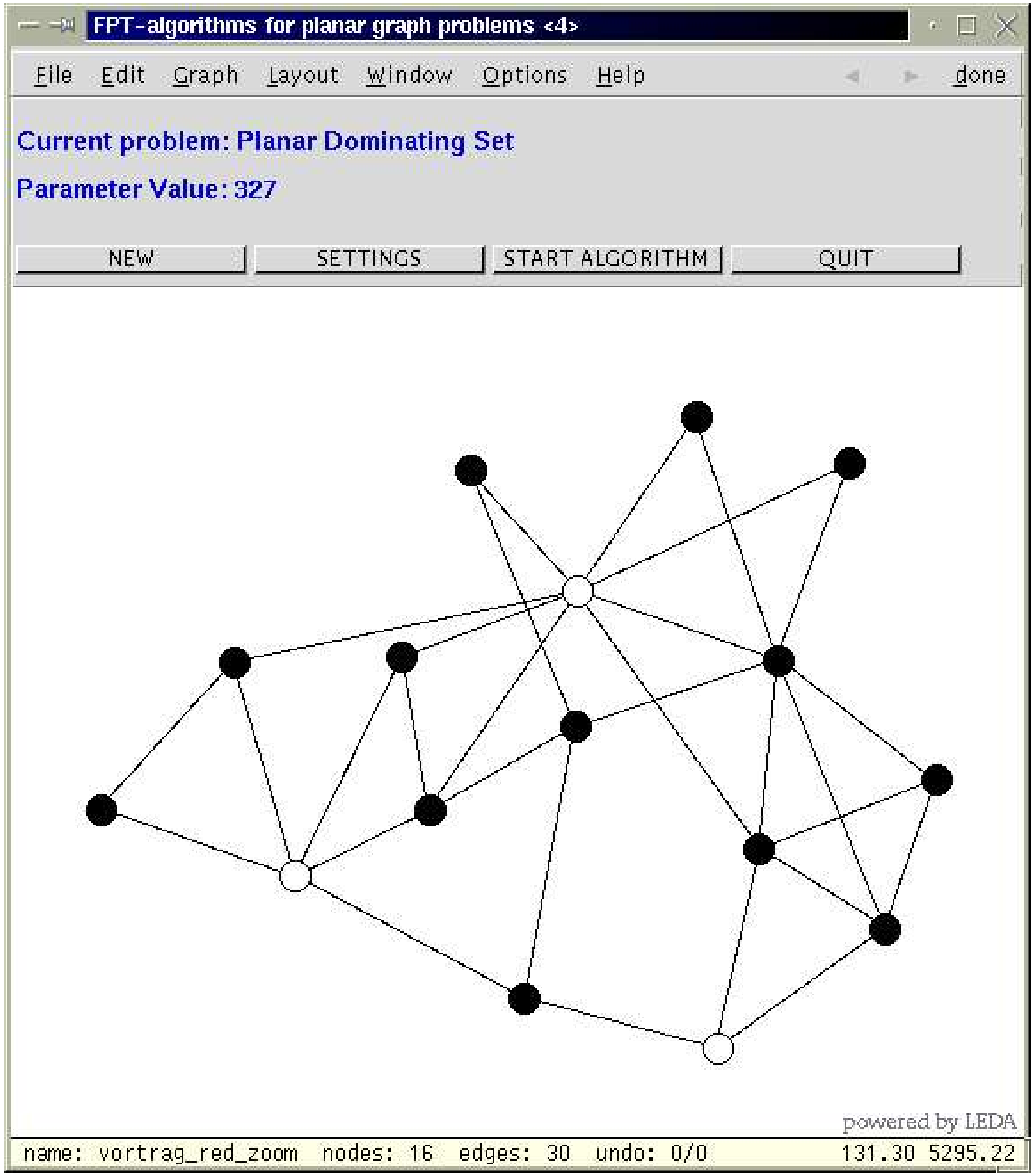

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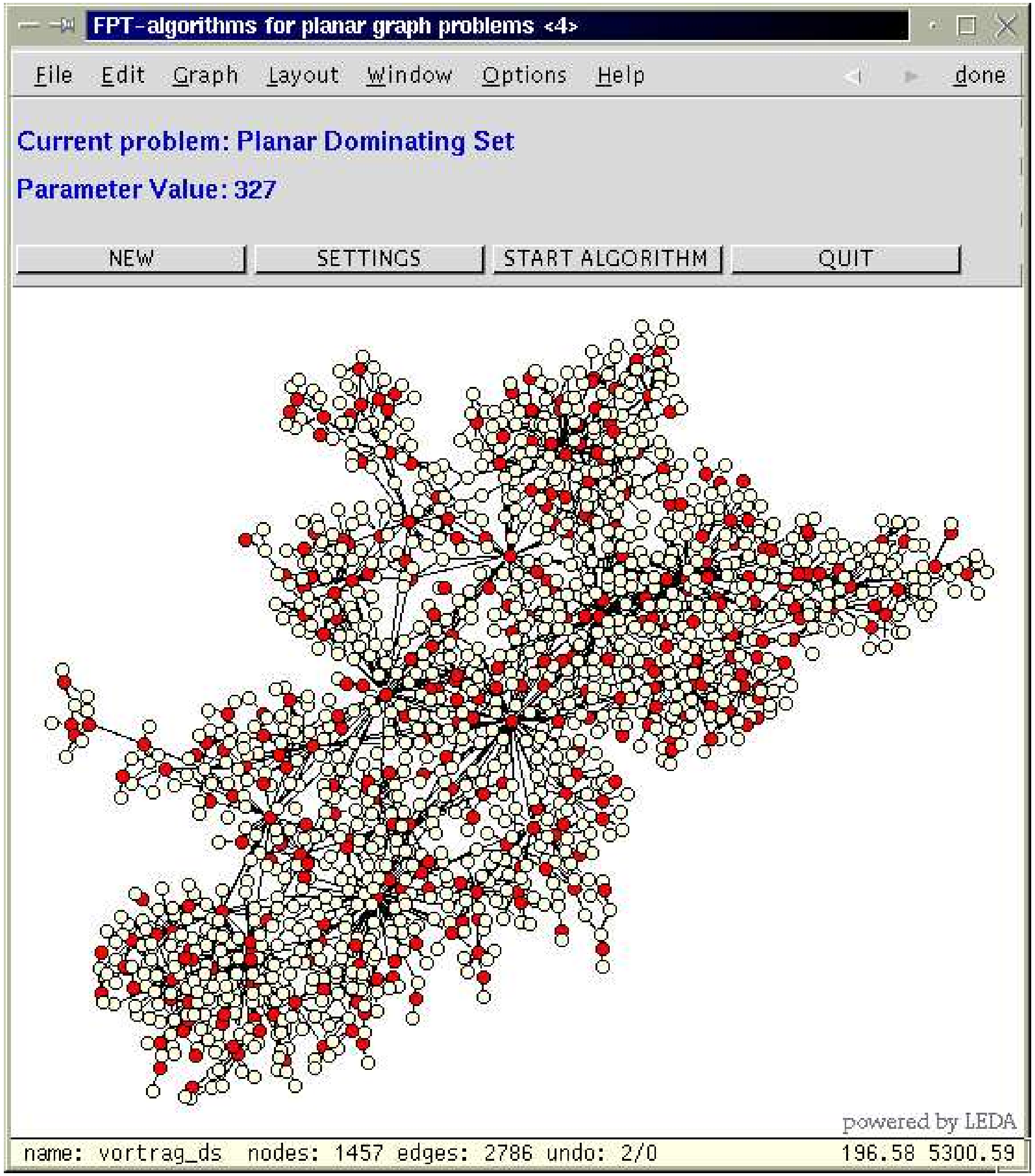


Experimental Results

Experimental Results

Autonomous Systems Graphs

[Chen et al., IEEE INFOCOM 2002]

	AS model: Oregon			
date	vertices	edges	% reduced	DS
03/31/01	10670	22002	100%	957
04/07/01	10729	21999	100%	969
04/14/01	10790	22469	100%	978
04/21/01	10895	22747	100%	982
04/28/01	10886	22493	100%	991
05/05/01	10943	22607	100%	988
05/12/01	11011	22677	100%	988
05/19/01	11051	22724	100%	979
05/26/01	11174	23409	100%	993

Autonomous Systems Graphs

[Chen et al., IEEE INFOCOM 2002]

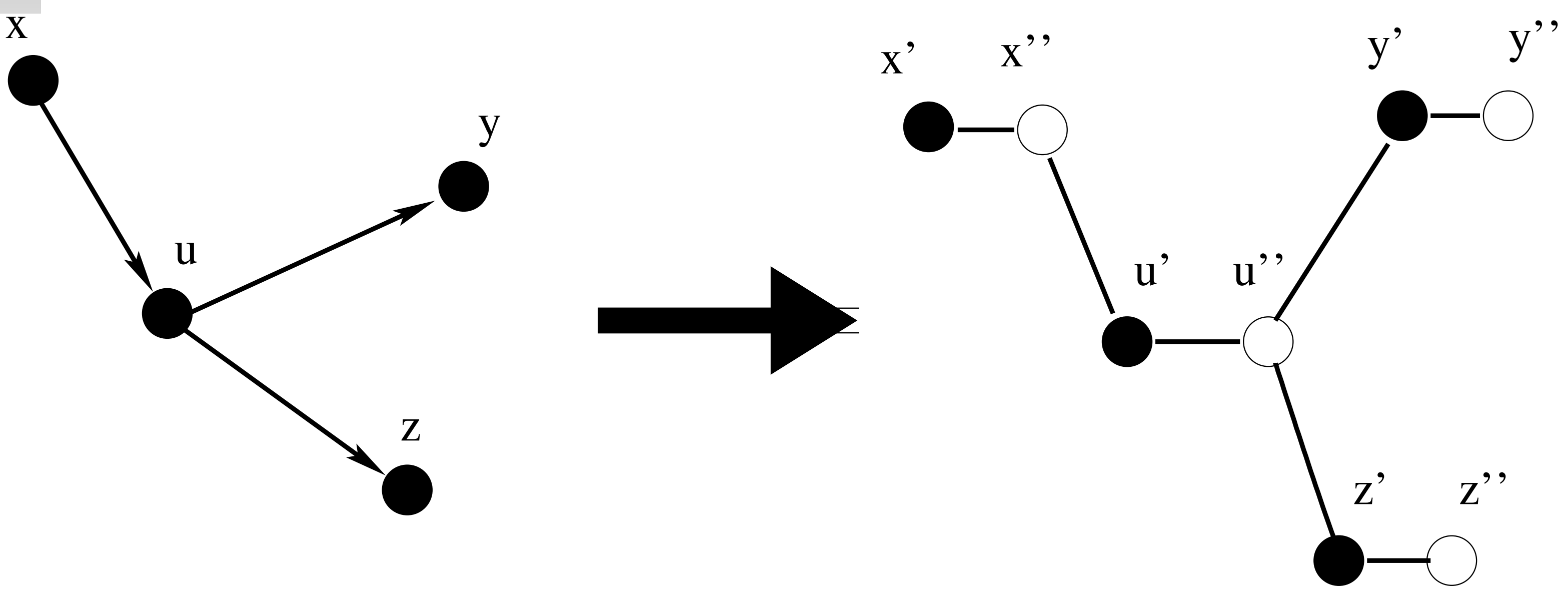
	AS model: Oregon				**enriched AS model: Oregon+**			
date	vertices	edges	% reduced	DS	vertices	edges	% reduced	DS
03/31/01	10670	22002	100%	957	10900	31180	100.00%	936
04/07/01	10729	21999	100%	969	10981	30855	99.97%	935
04/14/01	10790	22469	100%	978	11019	31761	99.92%	949
04/21/01	10895	22747	100%	982	11080	31538	99.95%	956
04/28/01	10886	22493	100%	991	11113	31434	100.00%	965
05/05/01	10943	22607	100%	988	11157	30943	99.89%	960
05/12/01	11011	22677	100%	988	11260	31303	99.89%	961
05/19/01	11051	22724	100%	979	11375	32287	99.90%	968
05/26/01	11174	23409	100%	993	11461	32730	99.92%	966

BRITE Topology Generator

[Medina et al. IEEE MASCOTS 2001]

	BRITE: 1000 vertices, 1997 edges				
	Type 1	Type 2	Type 3	Type 4	Type 5
# vertices removed (percentage)	1000 100%	668 64.8%	906 90.6%	915 91.5%	751 75.1%
# edges removed (percentage)	1997 100%	1450 72.6%	1873 93.8%	1892 94.7%	1558 78.0%
#vertices for DS found	195	120	173	176	146
time (sec)	77	127	67	71	87

Extension to Directed Networks



u' simulates that u needs to be dominated

u" simulates that u can dominate

Directed Networks

- **HTML network**
  - 739 vertices and 3447 arcs
  - Dominating Set of size 141 (less than 10 seconds)

  Document can be reached from 141 pages following only one link

- **food webs**
  - arcs from prey to predator
  - Dominating Sets of small sizes

  Animals of the Dominating Set affect menu of all predators

Conclusion and Outlook

bad news

behave poor applied to dense graphs with many edges [Sanchis]

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further experiments

extend the range of networks

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similar reduction rules for variants of Dominating Set

- Connected Dominating Set
- Power Dominating Set
- Perfect Dominating Set