

Multi-Player Diffusion Games on Graph Classes

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AKT



Scenario

- k players



Scenario

- k players
- playing on a graph on n vertices



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- playing on a graph on n vertices
- each player chooses a vertex



Scenario

- k players
- playing on a graph on n vertices
- each player chooses a vertex
- diffusion process

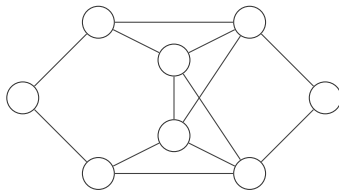


Scenario

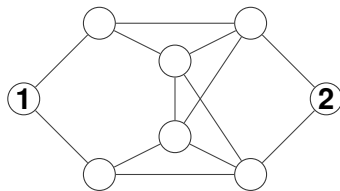
- k players
- playing on a graph on n vertices
- each player chooses a vertex
- diffusion process
- maximize number of vertices



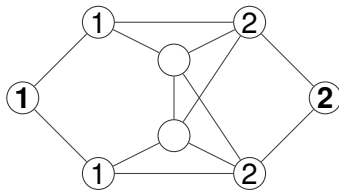
Example (two players on this input graph)



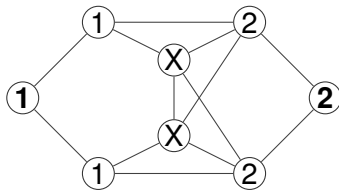
Example (strategy profile)



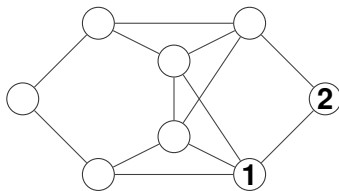
Example (diffusion step 1)



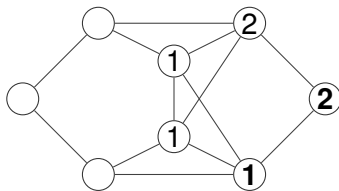
Example (diffusion step 2)



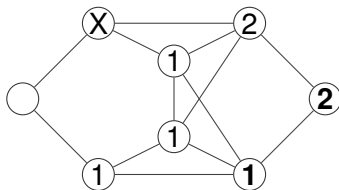
Player 1 can improve (different strategy profile)



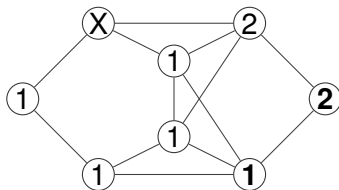
Player 1 can improve (diffusion step 1)



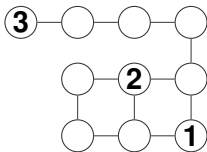
Player 1 can improve (diffusion step 2)



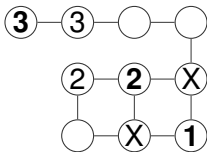
Player 1 can improve (diffusion step 3)



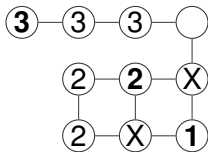
Don't call me Voronoi (Strategy profile)



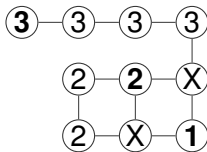
Don't call me Voronoi (diffusion step 1)



Don't call me Voronoi (diffusion step 2)



Don't call me Voronoi (diffusion step 3)



Companies competing over social networks

- Introduced by Alon et al. (IPL)



Companies competing over social networks

- Introduced by Alon et al. (IPL)
- Companies bribe people



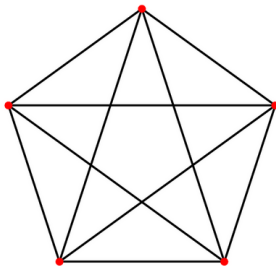
Companies competing over social networks

- Introduced by Alon et al. (IPL)
- Companies bribe people
- People persuaded by their friends (or get confused)



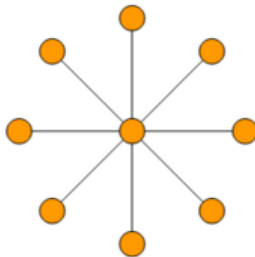
Already known

Nash exists for any k on any clique



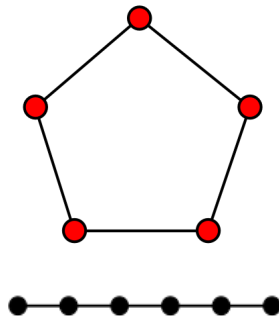
Already known

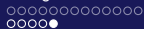
Nash exists for any k on any star



Already known

Nash exists for 2 on any path and on any cycle





What we do

- Nash for $k > 2$ on paths?



What we do

- Nash for $k > 2$ on paths?
- Nash for $k > 2$ on cycles?

What we do

- Nash for $k > 2$ on paths?
- Nash for $k > 2$ on cycles?
- Nash for $k = 3$ on grids?



What we do

- Nash for $k > 2$ on paths?
- Nash for $k > 2$ on cycles?
- Nash for $k = 3$ on grids?
- Nash for k on graphs on $f(k)$ vertices?

Even k on P_n

Lemma

*For any even $k \in \mathbb{N}$ and any $n \in \mathbb{N}$,
there is a Nash-equilibrium for k players on P_n .*

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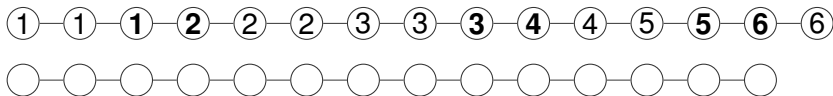
Lemma

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Odd $k > 3$ on P_n **Lemma**

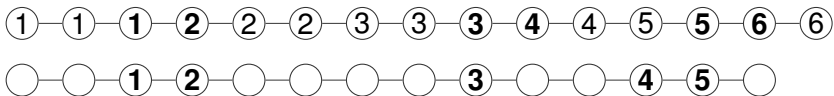
For any odd $k > 3 \in \mathbb{N}$ and for any $n \in \mathbb{N}$, there is a Nash-equilibrium for k players on P_n .



Odd $k > 3$ on P_n

Lemma

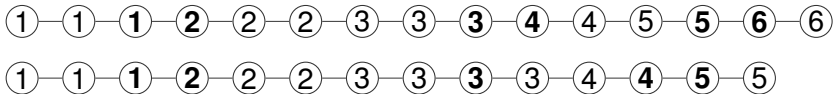
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Odd $k > 3$ on P_n

Lemma

For any odd $k > 3 \in \mathbb{N}$ and for any $n \in \mathbb{N}$, there is a Nash-equilibrium for k players on P_n .



Any k on C_n

- Close the path to a cycle

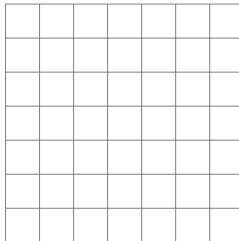
Any k on C_n

- Close the path to a cycle
- $k = 3$

$k = 3$

Overall

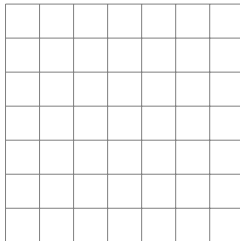
- play far



$k = 3$

Overall

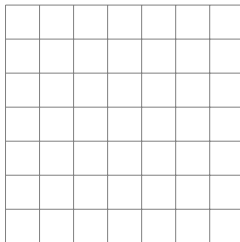
- play far
 - \exists strictly controls



$k = 3$

Overall

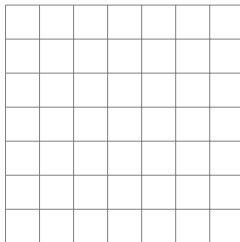
- play far
 - \exists strictly controls
 - \nexists strictly controls



$k = 3$

Overall

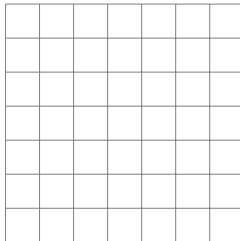
- play far
 - \exists strictly controls
 - \nexists strictly controls
- play close



$k = 3$

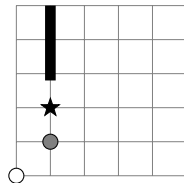
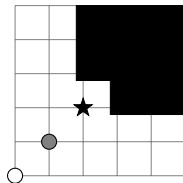
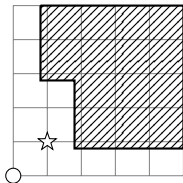
Overall

- play far
 - \exists strictly controls
 - \nexists strictly controls
- play close
 - case analysis



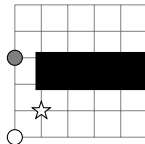
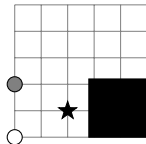
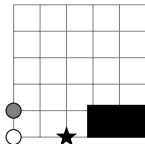
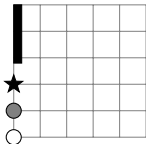
$k = 3$

Play far and \exists strictly controls



$k = 3$

Play far and ∇ strictly controls



$k = 3$

Play close

1



2



3



4



5



6



7



8



9



10

Two players

How many vertices are needed for \exists Nash-equilibrium?

Question

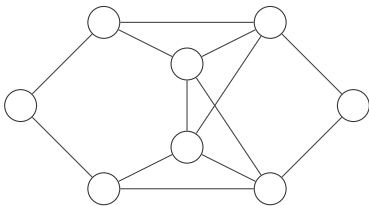
For two players, there is a Nash-equilibrium on each n -vertex graph if and only if $n \leq ???$

Two players

How many vertices are needed for \exists Nash-equilibrium?

Answer

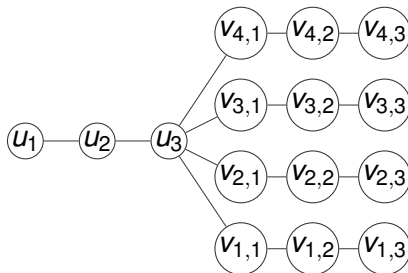
For two players, there is a Nash-equilibrium on each n -vertex graph if and only if $n \leq 7$.



How many vertices are needed for \exists Nash-equilibrium?

Theorem

For any $k > 2$ and any $n \geq \lfloor \frac{3}{2}k \rfloor + 2$, there exists a tree with n vertices such that there is no Nash-equilibrium for k players.



What we have done

- Paths and cycles

What we have done

- Paths and cycles
- Three players on grids

What we have done

- Paths and cycles
- Three players on grids
- General graphs

What we haven't done

- More than three players on grids

What we haven't done

- More than three players on grids
- Does $n \leq \frac{3}{2}k + 1$ implies \exists Nash-equilibrium?

Prologue
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Paths and Cycles
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○○○
○

Grids
○○○○

General graphs
○○
○

Epilogue
○
○
●

Q?

Q?

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